DESIGN

OF

REINFORCED CEMENT

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CHAPTER 1 GENERAL

1.1 SYMBOLS

For the purpose of this standard, the following letter symbols shall have the meaning indicated against each; where other symbols are used, they are explained at the appropriate place:

- A : Area
- b : Breadth of beam, or shorter dimension of a rectangular column
- bef : Effective width of slab
- bef : Effective width of flange
- b'w : Breadth of web or rib
- D : Overall depth of beam or slab or diameter of column; dimension of a rectangular column in the direction under consideration.
- D t : Thickness of flange
- DL : Dead Load
- d : Effective depth of beam or slab
- d' : Depth of compression reinforcement from the highly compressed face
- Ec : Modulus of elasticity of concrete
- EL : Earthquake load
- Es : Modulus of elasticity of steel
- e : Eccentricity
- f ck : Characteristic cube compressive strength of concrete
- f cr : Modulus of rupture of concrete (flexural tensile strength)
- f ct : Splitting tensile strength of concrete
- f d : Design strength
- f y : Characteristic strength of steel
- H w : Unsupported height of wall
- H we : Effective height of wall
- lef : Effective moment of inertia
- I gr : Moment of inertia of the gross section excluding reinforcement
- I r : Moment of inertia of cracked section
- K : Stiffness of member
- k : Constant or coefficient or factor
- L d : Development length
- LL : Live load or imposed load
- Lw : Horizontal distance between centers of lateral restraint
- l : Length of a column or beam between adequate lateral restraints or the unsupported length of a column
- l ef : Effective span of beam or slab or effective length of column
- l ex : Effective length about x-x axis
- l ey : Effective length about y-y axis
- l n : Clear span, face-to-face of supports
- l'n : l' for shorter of the two spans at right angles
- l x : Length of shorter side of slab
- l y : Length of longer side of slab
- l o : Distance between points of zero moments in a beam
- l t : Span in the direction in which moments are determined, center to center of supports.
- l 2 : Span transverse to l 1 center to center of supports
- l' 2 : l 2 for the shorter of the continuous spans
- M : Bending moment
- m : Modular ratio
- n : Number of samples
- P : Axial load on a compression member
- q o : Calculated maximum bearing pressure of soil
- r : Radius
- s : Spacing of stirrups or standard deviation
- T : Torsional moment
- t : Wall thickness
- V : Shear force
- W : Total load
- W L : Wind load
- w : Distributed load per unit area
1.2 MATERIALS

1.2.1 Cement (Clause 5.1 – IS: 456/2000)

The cement used shall be any of the following and the type selected should be appropriate for the intended use:
   a) 33 Grade ordinary Portland cement conforming to IS 269
   b) 43 Grade ordinary Portland cement conforming to IS 8112
   c) 53 Grade ordinary Portland cement conforming to IS 12269
   d) Portland slag cement conforming to IS 455

1.2.2 Aggregate (Clause 5.3 – IS: 456/2000)

For most work, 20mm aggregate is suitable. Where there is no restriction to the flow of concrete into sections, 40 mm or larger size may be permitted. For thin sections - 10 mm.

For heavily reinforced concrete members as in the case of ribs of main beams, the nominal maximum size of the aggregate should usually be restricted to 5mm less than the minimum clear distance between the main bars or 5 mm less than the minimum cover to the reinforcement whichever is smaller.

1.2.3 Water (Clause 5.4 – IS: 456/2000)

pH value shall not be less than 6
1.2.4 Admixtures (Clause 5.5 – IS: 456/2000)

Admixture, if used shall comply with IS: 9103.

1.2.5 Reinforcement (Clause 5.6 – IS: 456/2000)

Mild steel IS: 432 (Part – I) High strength deformed steel bars conforming to IS: 1786. Es of steel (Modulus of elasticity) = 200 kN / mm²

CHAPTER 2 CONCRETE

2.1 Grades Designation of concrete : M20

M refers to the mix and 20 refers to the specified compressive strength of 150 mm size cube at 28 days expressed in N / mm²

2.2 Tensile strength

Flexural strength fcr = 0.7 \( \sqrt{f_{ck}} \) N / mm²

fck = Characteristic cube compressive strength of concrete in N / mm²

2.3 Modulus of Elasticity

\[ Ec = 5000 \sqrt{f_{ck}} \text{ N / mm}^2 \]

Actual measured values may differ by (+) or (-) 20 percent from the values obtained from the above expression.

2.4 Shrinkage

Approximate value of the total shrinkage strain for design may be taken as 0.0003 (Refer IS: 1343)

2.5 Creep of concrete (IS : 456 / 6.2.5.1)

<table>
<thead>
<tr>
<th>Age at loading</th>
<th>Creep coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 days</td>
<td>2.2</td>
</tr>
<tr>
<td>28 days</td>
<td>1.6</td>
</tr>
<tr>
<td>1 year</td>
<td>1.1</td>
</tr>
</tbody>
</table>

2.6 Workability (IS : 456 / 7)
The concrete mix proportions chosen should be such that the concrete is of adequate workability for the placing conditions of the concrete and can properly be compacted with the means available. Suggested ranges of workability of concrete measured in accordance with IS 1199 are given below:

**Table 2.1 Suggested ranges of workability of concrete measured in accordance with IS 1199**

<table>
<thead>
<tr>
<th>Placing Conditions</th>
<th>Degree of workability</th>
<th>Slump (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blinding concrete; Shallow sections; Pavements using pavers</td>
<td>Very low</td>
<td>*</td>
</tr>
<tr>
<td>Mass concrete; Lightly reinforced sections in slabs, beams, walls, columns; Floors; Hand placed pavements; Canal lining; Strip footings</td>
<td>Low</td>
<td>25 – 75</td>
</tr>
<tr>
<td>Heavily reinforced sections in slabs, beams, walls, columns; Slipform work; pumped concrete</td>
<td>Medium</td>
<td>50 – 100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>75 – 100</td>
</tr>
<tr>
<td>Trench fill; In-situ piling</td>
<td>High</td>
<td>100 – 150</td>
</tr>
<tr>
<td>Tremie concrete</td>
<td>Very high</td>
<td>**</td>
</tr>
</tbody>
</table>

**Note:** For most of the placing conditions, internal vibrators (needle vibrators) are suitable. The diameter of the needle shall be determined based on the density and spacing of reinforcement bars and thickness of sections. For tremie concrete, vibrators are not required to be used

*In the ‘very low’ category of workability where strict control is necessary, for example pavement quality concrete, measurement of workability by determination of compacting factor will be more appropriate than slump (see IS 1199) and a value of compacting factor of 0.75 to 0.8 is suggested.

**In the ‘very high’ category of workability, measurement of workability by determination of flow will be appropriate (See IS 9103)**

**Table 2.2 Minimum cement content, Maximum Water - cement Ratio and minimum grade of concrete for different exposures with normal weight aggregates of 20 mm nominal maximum size.**

<table>
<thead>
<tr>
<th>SI No.:</th>
<th>Exposure</th>
<th>Minimum concrete</th>
<th>Reinforced concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4) (5) (6)</td>
</tr>
<tr>
<td>i</td>
<td>Mild</td>
<td>220</td>
<td>0.60</td>
</tr>
<tr>
<td>ii</td>
<td>Moderate</td>
<td>240</td>
<td>0.60 M15</td>
</tr>
<tr>
<td>iii</td>
<td>Severe</td>
<td>250</td>
<td>0.50 M20</td>
</tr>
<tr>
<td>iv</td>
<td>Very severe</td>
<td>260</td>
<td>0.45 M20</td>
</tr>
<tr>
<td>v</td>
<td>Extreme</td>
<td>280</td>
<td>0.40 M25</td>
</tr>
</tbody>
</table>

**Notes:**
1. Cement content prescribed in this table is irrespective of the grades of cement and it is inclusive of additions mentioned in 5.2 (see IS: 456). The additions such as flyash or ground granulated blast furnace slag may be taken into account in the concrete composition with respect to the cement content and water cement ratio if the suitability is established and as long as the maximum amounts taken into account do not exceed the limit of pozzolona and slag specified in IS 1489 (part-I) and IS 455 respectively.
2. Minimum grade for plain concrete under mild exposure condition is not specified.
Table 2.3 Proportion for Nominal Mix concrete

<table>
<thead>
<tr>
<th>Grade of concrete</th>
<th>Total quantity of dry aggregates by mass per 50kg of cement, to be taken as the sum of the individual masses of fine and coarse aggregates, kg, max</th>
<th>Proportion of fine Aggregate to coarse Aggregate (by Mass)</th>
<th>Quantity of water per 50 kg of cement, max</th>
</tr>
</thead>
<tbody>
<tr>
<td>M5</td>
<td>800</td>
<td>Generally 1:2 but subject to an upper limit of 1:1 ½ and a lower limit of 1:2 ½</td>
<td>60</td>
</tr>
<tr>
<td>M7.5</td>
<td>625</td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>M10</td>
<td>480</td>
<td></td>
<td>34</td>
</tr>
<tr>
<td>M15</td>
<td>330</td>
<td></td>
<td>32</td>
</tr>
<tr>
<td>M20</td>
<td>250</td>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>

Note: The proportion of the fine to coarse aggregates should be adjusted from upper limit to lower limit progressively as the grading of fine aggregates becomes finer and the maximum size of coarse aggregate becomes larger. Graded coarse aggregate shall be used.

2.7 Stripping time of form work

While the criteria of strength shall be the guiding factor for removal of form work, in normal circumstances where ambient temperature does not fall below 15 °C and where ordinary Portland cement is used and adequate curing is done, following striking period may deem to satisfy the guideline.

Table 2.4 De shuttering period

<table>
<thead>
<tr>
<th>Type of form work</th>
<th>Minimum period before striking form work</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Vertical form work to columns, walls, beams</td>
<td>16 – 24 h</td>
</tr>
<tr>
<td>(b) Soffit form work to slabs (props to be fixed again immediately after removal of form work)</td>
<td>3 days</td>
</tr>
<tr>
<td>(c) Soffit form work to beams (props to be fixed again immediately after removal of form work)</td>
<td>7 days</td>
</tr>
<tr>
<td>(d) props to slabs: 1) spanning up to 4.5m 2) spanning over 4.5m</td>
<td>7 days 14 days</td>
</tr>
<tr>
<td>(e) props to beams and arches: 1) spanning up to 6m 2) spanning over 6m</td>
<td>14 days 21 days</td>
</tr>
</tbody>
</table>

For other cements and lower temperature, the stripping time recommended above may be suitably modified.

2.8 Construction joints

As per IS: 11817.

CHAPTER 3 ASSEMBLY OF REINFORCEMENT (AS PER IS 2502)

3.1 Tolerances on placing reinforcement

Unless specified otherwise by engineer – in – charge, the reinforcement shall be placed within the following tolerances:

a) For effective depth 200 mm or less ± 10 mm
b) For effective depth more than 200 mm ± 15 mm

3.2 Tolerance for cover
Unless specified otherwise, actual concrete cover should not deviate from the required nominal cover by +10 mm -0 mm

Nominal cover as given in 26.4.1/IS 456:2000 should be specified to all steel reinforcement including links. Spacers between the links (or the bars where to links exist) and the formwork should be of the same nominal size as the nominal cover.

Spacers, chairs and other supports detailed on drawings, together with such other supports as may be necessary, should be used to maintain the specified nominal cover to the steel reinforcement. Spacers or chairs should be placed at a maximum spacing of 1 m and closer spacing may sometimes be necessary.

Spacers, cover blocks should be of concrete of same strength or PVC.

CHAPTER 4 GENERAL DESIGN CONSIDERATION

4.1 Loads and forces (Clause 19 – IS: 456/2000)

Forces or other actions that reset from the weight of all building materials, occupants and their possessions, environmental effects, differential movements and restrained dimensional changes.

4.1.1 Dead loads

Dead Loads consist of the weight of walls, partitions, floors, floor finish, roofing and all permanent constructions (Refer IS 875 Part 1 for unit weight) and weights of all materials of construction incorporated into the building including stair-ways, built in partitions, finishes, cladding and other similarity incorporated architectural and structural items and fixed service equipment items, and fixed service equipment including the weight of cranes

- As per IS : 875 (Part 1)
- Plain cement concrete 24 kN / m³
- Reinforced cement concrete 25 kN / m³

4.1.2 Imposed loads
Imposed Loads are those loads produced by the use and occupancy of the building or other structure and do not include construction of environmental loads such as wind loads, snow loads, rain load, earth – quake load, flood load (or) dead load. Live load on a roof are those produced as follows. During maintenance by workers, equipment and materials and during the life of the structure by movable objects such as planters and by people. This shall be as per IS: 875 (Part 2)

4.1.3 Wind loads

Air in motion is called wind. When the motion is obstructed by the building, wind force (load) is imparted. This acts laterally on the building. As per IS: 875 (Part 3)

4.1.4 Snow loads

As per IS: 875 (Part 4)

4.1.5 Earthquake forces (seismic forces)

Ground in motion is called Seismic or Earth Quake which shakes the building. This can be in any direction, but lateral load is predominant. As per IS: 1893.

4.1.6 Shrinkage, creep and temperature effects

Refer Clause 6.2.4, 6.2.5, 6.2.6 of IS: 456/2000 and IS: 875 Part – 5

In ordinary buildings, such as low rise dwellings whose lateral dimension do not exceed 45m, the effects due to temperature fluctuations and shrinkage and creep can be ignored in design calculations.

4.1.7 Other forces and effects.

- Foundation movement (IS: 1904)
- Elastic axial shortening
- Soil and fluid pressures IS: 875 (Part – 5)
- Vibration
- Fatigue
- Impact IS: 875 (Part 5)
- Erection loads IS: 875 (Part 2)
- Stress concentration effect due to point load and the like.

4.1.8 Combination of loads

As in IS: 875 (Part 5)

4.1.9 Allowable Stress Design:

A method of proportioning structural members such that elastically computed stress produced in the members by nominal loads do not exceed specified allowable stresses (also called working stress design)

4.1.10 Limit State:

A condition beyond which a structure or member becomes unfit for service and is judged either to be no longer useful for its intended function. (Serviceability limit state) or to be unsafe (Strength Limit State)

4.1.11 Factored Load

The product of the nominal load and a load factor.
4.1.12 P-Delta Effect:-

The second order effect on shear and moments of frame members induced by axial loads on a laterally displaced building frame.

4.1.13 Modular ratio:-

The simple theory of bending is applicable also to concrete beams, and, from the assumption that plane sections remain plane after bending, it follows that the strains in the material at various distances from the neutral axis are proportional to these distances. In a reinforced concrete beam, the steel and concrete are well bonded together, hence the strains in both of the materials will be equal at the center line of the steel reinforcement.

Since concrete is weak in tension, it is assumed that the concrete below NA is ineffective and the tensile load is taken up by steel.

Strain at top of concrete = \( e_c = A \frac{B}{A_B} \quad (1) \)

Strain in concrete surrounding steel = \( e_t = c - AB \quad (2) \)

The strain in the steel is same as in concrete surrounding it, and is therefore equal to \( e_t \). The stresses across the section (before the material is cracked) are equal to the corresponding strain multiplied by the modulus of elasticity.

Stress at top of concrete = \( \sigma_{cbc} = e_c \times \frac{E_c}{E_s} \quad (1) \)

Similarly the stress in the steel is = \( \sigma_{st} = e_t \times \frac{E_s}{E_c} \quad (2) \)

If the concrete is not cracked, then the stress in concrete surrounding the steel would be

\( \sigma_{cbct} = e_t \times \frac{E_c}{E_s} \quad (3) \)

From equation 2, the stress in steel is

\( \sigma_{st} = e_t \times \frac{E_c}{E_s} \times \frac{E_s}{E_c} \times \sigma_{cbct} = m \times \sigma_{cbct} \)

\( m \) is the modular ratio.

Therefore \( \sigma_{st} = m \sigma_{cbct} \)

\( \sigma_{cbct} = \sigma_{st} / m \)

Stress in steel = stress in concrete \( \times m \)

Modular ratio:
The ratio of the moduli of elasticity of the different materials of a composite member is

\[ m = \frac{280}{3\sigma_{c,b}} \]

Where \( \sigma_{c,b} \) is the permissible compressive stress due to bending in concrete in N/mm\(^2\).

**Equivalent area:**

In the case of bending also, the strain in steel will be equal to the strain in the surrounding concrete.

\[ E_S = E_C \]

\[ \frac{\sigma_S}{E_S} = \frac{\sigma_C}{E_C} \]

\[ \sigma_S = \frac{E_S}{E_C} \times \sigma_C = m\sigma_C \]

IS : 456 suggests to permit a compressive stress in steel equal to 1.5 times ‘m’ the compressive stress in the surrounding concrete in flexural members.

**4.1.14 Moment distribution method**

‘K’ is the beam stiffness required to produce unit rotation. ‘M’ is the moment required to produce unit stiffness.

\[ \mu = K\theta \]

\[ \theta_B = 1 \quad K = 4EI/\ell \]

\[ \theta_B = 1 \quad K = 3EI/\ell \]

The stiffness of a simply supported beam is \( \frac{3}{4} \) of the stiffness of the same beam when it is fixed at one end and S.S (or hinged) at the other end.

Total moment = \( \mu_1 + \mu_2 + \mu_3 + \mu_4 \)

Total stiffness = \( K_1 + K_2 + K_3 + K_4 \)

\[ \mu_1 = K_1\theta \]

\[ \mu_2 = K_2\theta \]

\[ \mu_3 = K_3\theta \]

\[ \mu_4 = K_4\theta \]

\[ \mu_1/\mu = K_1/\mu \quad K = K_1/\mu \quad \& \quad \mu = (K_1/\mu)/K \]

\[ \mu_2/\mu = K_2/\mu \quad \& \quad \mu_2 = (K_2/\mu)/K \]

\[ \mu_3/\mu = K_3/\mu \quad \& \quad \mu_3 = (K_3/\mu)/K \]

\[ \mu_4/\mu = K_4/\mu \quad \& \quad \mu_4 = (K_4/\mu)/K \]

The quantities \( K_1/K, K_2/K, K_3/K, K_4/K \) are called Distribution factors.

Distribution factor of OA = \( K_1/K \).

The moment distributed to the member OA = the applied moment x Distribution factor (ie) \( K/\Sigma K \mu \) is the applied moment.

**4.1.15 Columns subjected to bending moments:**

The exact distribution of the bending moments among the different members at a joint can be done by moment distribution, slope deflection on any other accepted methods of analysis. An approximate method of analysis is to distribute the fixed moment at the joint in the ratio of stiffness of the member to the total stiffness of all members meeting at the joint.
Lower Column.

The moment in the lower column.

\[ M = \left( \frac{kl}{k_u + kl + k_b} \right) x M_f \quad ...... \quad (1) \]

- \( M_1 \) = fixed moment in beam 1
- \( M_2 \) = fixed moment in beam 2
- Difference in moment = \( M_0 = M_1 - M_2 \)
- \( K_u = \) Stiffness of upper column = \( I_u / L_u \)
- \( K_l = \) Stiffness of lower column = \( I_l / L_L \)
- \( K_b = \) Stiffness of beam column = \( I_B / L_B \)

If the beam is subjected to uniformly distributed load of \( w \) kN / m and \( \ell \) is the Span in meters.

\[ M_f = \frac{wl^2}{12} \quad ...... \quad (2) \]

- Stiffness of member fixed at both ends = \( l / \ell \)
- Stiffness of a member fixed at one end and simply supported at the other end = \( (3/4) \times (l/l) \)
- Stiffness of a member simply supported at both the ends = \( l / 2\ell \)

Where \( I \) = moment of inertia of the member in mm\(^4\); \( \ell \) = length of the member in mm

(Unsupported length can be considered for the column)

If the beam is not restrained fully, the total stiffness = \( k_u + k_l + k_b/2 \)

(Where \( k_u = (l / l) \) of beam)

Note that half the stiffness of the beam only is considered if the beam is not restrained.

CHAPTER 5 STABILITY OF THE STRUCTURE (CLAUSE 20 OF IS :456/2000)

5.1 Overturning

The stability of a structure as a whole against overturning shall be ensured so that the restoring moment shall be not less than the sum of 1.2 times the maximum overturning moment due to the characteristic dead load and 1.4 times the maximum overturning moment due to the characteristic imposed
loads. In cases where dead load provides the restoring moment, only 0.9 times the characteristic dead load shall be considered. Restoring moment due to imposed loads shall be ignored.

5.2 Sliding

The structure shall have a factor against sliding of not less than 1.4 under the most adverse combination of the applied characteristic forces. In this case only 0.9 times the characteristic dead load shall be taken into account.

5.3 Lateral sway

Under transient wind load the lateral sway at the top should not exceed $H / 500$, where $H$ is the total height of the building for seismic load refer – IS: 1893.

CHAPTER- 6 ANALYSES AND DESIGN (CLAUSE OF IS: 456/2000)

6.1 Effective span (Clause 22.2 of IS: 456/2000)

Unless otherwise specified, the effective span of a member shall be as follows:

(a) Simply supported beam or slab – The effective span of a member that is not built integrally with its supports shall be taken as clear span plus the effective depth of slab or beam or centre to centre of supports, whichever is less.

(b) Continuous beam or slab – In the case of continuous beam or slab, if the width of the support is less than 1/12 of the clear span, the effective span shall be as in 6.1 (a). If the supports are wider than 1/12 of the clear span or 600 mm whichever is less, the effective span shall be taken as under:

1) For end span with one end fixed and the other continuous or for intermediate spans, the effective span shall be the clear span between supports;
2) For end span with one end free and the other continuous, the effective span shall be equal to the clear span plus half the effective depth of the beam or slab or the clear span plus half the width of the discontinuous support, whichever is less;
3) In the case of spans with roller or rocket bearings, the effective span shall always be the distance between the centres of bearings.

(c) Cantilever – The effective length of a cantilever shall be taken as its length to the face of the support plus half the effective depth except where its forms the end of a continuous beam where the length to the centre of support shall be taken.

(e) Frames – In the analysis of a continuous frame, centre to centre distance shall be used.

6.2 Arrangement of Imposed load (Clause 22.4.1 of IS: 456/2000)

a) Consideration may be limited to combinations of:
   1) Design dead load on all spans with full design imposed load on two adjacent spans; and
   2) Design dead load on all spans with full design imposed load on alternate spans.

b) When design imposed load does not exceed three – fourths of the design dead load, the load arrangement may be design dead load and design imposed load on all the spans.

Note: For beams and slabs continuous over support 6.2(a) may be assumed.

6.3 Moment and shear coefficients for continuous beams (Clause 22.5 of IS:456/2000)

Unless more exact estimates are made, for beams of uniform cross-section which support substantially uniformly distributed loads over three or more spans which do not differ by more than 15 percent of the longest, the bending moments and shear forces used in design may be obtained using the coefficients given in Table 6.1 and Table 6.2 respectively.

For moments at supports where two unequal spans meet or in case where the spans are not equally loaded, the average of the two values for the negative moment at the support may be taken for design.

Where coefficients given in Table 6.1 are used for calculation of bending moments, redistribution referred to in 6.6 shall not be permitted.

6.3.1 Beams and slabs over Free End supports

Where a member is built into a masonry wall which develops only partial restraint, the member shall be designed to resist a negative moment at the face of the support of \( Wl/24 \) where \( W \) is the total design load and \( l \) is the effective span, or such other restraining moment as may be shown to be applicable. For such a condition shear coefficient given in table 6.2 at the end support may be increased by 0.05.

Table 6.1 Bending Moment Coefficients

<table>
<thead>
<tr>
<th>Type of load</th>
<th>Span moments</th>
<th>Support moments</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>Near Middle of End span</td>
<td>At middle of interior span</td>
<td>At support Next to the End support</td>
</tr>
<tr>
<td>Dead load and imposed load (fixed)</td>
<td>+1/12</td>
<td>+1/16</td>
<td>-1/10</td>
</tr>
<tr>
<td>Imposed load (Not fixed)</td>
<td>+1/10</td>
<td>+1/12</td>
<td>-1/9</td>
</tr>
</tbody>
</table>

Note: For obtaining the bending moment, the coefficient shall be multiplied by the total design load and effective span.

Table 6.2 Shear force Coefficients

<table>
<thead>
<tr>
<th>Type of load</th>
<th>At End support</th>
<th>At support Next to the End support</th>
<th>At All Other Interior supports</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>Outer side</td>
<td>Inner side</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
</tbody>
</table>
Dead load and imposed load (fixed)  | 0.4  | 0.6  | 0.55  | 0.5  
Imposed load (not fixed)       | 0.45 | 0.6  | 0.6   | 0.6   

**Note:** For obtaining the shear force, the coefficient shall be multiplied by the total design load.

### 6.4 Critical sections for moment and shear (Clause 22.6.1 of IS: 456/2000)

For monolithic construction, the moments computed at the face of the supports shall be used in the design of the members at these sections. For non-monolithic construction the design of the member shall be done keeping in view (6.1 above – IS 456 / 22.2).

### 6.5 Critical Section for shear (Clause 22.6.2 of IS:456/2000)

The shears computed at the face of the support shall be used in the design of the member at that section except as in 6.5.1.

6.5.1 When the reaction in the direction of the applied shear introduces compression into the end region of the member, sections located at a distance less than \(d\)' from the face of the support may be designed for the same shear as that computed at distance \(d\) (see Fig. 2).

**Note:** The above clauses are applicable for beams generally carrying uniformly distributed load or where the principle load is located farther than \(2d\) from the face of the support.

![Diagram](image)

### 6.6 Redistribution of moments (clause 22.7 of IS: 456/2000)

Redistribution of moments may be done in accordance with 37.1.1/IS 456:2000 for limit state method and in accordance with B-1.2/ IS 456:2000 for working stress method. However, where simplified analysis using coefficients is adopted, redistribution of moments shall not be done.

### 6.7 Design methods:

- Working Stress.
- Limit state.
- Experimental Investigations.

#### 6.7.1 Limit state method:

In limit state method of design, the design strength of steel is taken as

- \(0.87f_y\) in tension & shear.
- \(0.67f_y\) in direct compression.

Where \(f_y\) is the characteristic strength of steel.

- Limit State of Collapse (or) Ultimate limit State
- Limit State of Serviceability.
6.7.2 Assumptions in limit state method (in flexure):

- Plane sections normal to the axis remain plane after bending.
- The maximum strain in concrete at the outermost compression fiber is taken as 0.0035 in bending.
- The Compressive strength of concrete is assumed as 0.67fck and the design strength of concrete is assumed as 0.446fck where fck is the characteristic strength of concrete. The compressive force in concrete is taken as 0.36fck x xu acting at a depth of 0.42xu where xu is the depth of neutral axis.
- Tensile Strength of concrete is ignored.
- The tensile strength of steel is taken as fy and the design strength of steel is assumed as 0.87fy where fy is the characteristic strength of steel.
- The maximum strain in the tension reinforcement in the section at failure shall not be less than

\[
\left( \frac{ fy }{ 0.025 \times 0.15 } \right)_{\text{occ}}
\]

6.7.3 Load factor:

The margin of safety against failure of a structure is referred to as a load factor and defined as a ratio of failure load to working load.

IS: 456 recommends following load factors.

(a) For structures in which the effect of wind and earthquake load is negligible

\[ U = 1.5D.L + 2.2L.L \]

(b) When wind load is to be considered

\[ U = 1.5D.L + 2.2L.L + 0.5W.L \] (or) \[ U = 1.5D.L + 0.5L.L + 2.2W.L \]

Whichever gives the critical condition, provided that no member shall have a capacity less than required by the condition (a)

(c) For structure subjected to earthquakes

\[ U = 1.5D.L + 2.2L.L + 0.5E.L \] (or) \[ U = 1.5D.L + 0.5L.L + 2.2E.L \]

Whichever gives the critical condition.

6.8 Ultimate Strength in Flexure

Strength of concept in tension is neglected both in working stress and ultimate strength. Average compression stress in the stress block = 0.55\( \sigma_{cu} \) and depth is 0.75n. To ensure primary tension failure, the depth is limited to 0.43d. The balanced M.R = 0.185bd\( \sigma_{cu} \) and the balanced tension reinforcement index is \( q_b = A_t / bd \cdot \sigma_{sy} / \sigma_{cu} = 0.236 \).
CHAPTER 7 SOLID SLABS

7.1 General

The provisions of 7.2 for beams apply to slabs also.

Notes:
For slabs spanning in two directions, the shorter of the two spans should be used for calculating the span to effective depth ratios.
For two-way slabs of shorter spans (up to 3.5m) with mild steel reinforcement, the span to overall depth ratios given below may generally be assumed to satisfy vertical deflection limits for loading class up to 3 kN/m²

<table>
<thead>
<tr>
<th>Slab Type</th>
<th>Span to Overall Depth Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply supported slabs</td>
<td>35</td>
</tr>
<tr>
<td>Continuous slabs</td>
<td>40</td>
</tr>
</tbody>
</table>

For high strength deformed bars of grade Fe415, the values given above should be multiplied by 0.8.
(Span to over all depth ratio for simply supported slabs spanning in one direction is 30. For cantilever slabs, it is 12)

7.2 Slabs Continuous Over Supports

Slabs spanning in one direction and continuous over supports shall be designed according to the provisions applicable to continuous beams.

7.3 Slabs Monolithic With Supports

Bending moments in slabs (except flat slabs) constructed monolithically with the supports shall be calculated by taking such slabs either as continuous over supports and capable of free rotation, or as members of a continuous framework with the supports, taking into account the stiffness of such supports. If such supports are formed due to beams which justify fixity at the support of slabs, then the effects on the supporting beam, such as, the bending of the web in the transverse direction of the beam and the torsion in the longitudinal direction of the beam, wherever applicable, shall also be considered in the design of the beam.

7.3.1 For the purpose of calculation of moments in slabs in a monolithic structure, it will generally be sufficiently accurate to assume that members connected to the ends of such slabs are fixed in position and direction at the ends remote from their connections with the slabs.

7.3.2 Slabs carrying concentrated load

7.3.2.1 If a solid slab supported on two opposite edges, carries concentrated loads the maximum bending moment caused by the concentrated loads shall be assumed to be resisted by an effective width of slab (measured parallel to the supporting edges) as follows:

a) For a single concentrated load, the effective width shall be calculated in accordance with the following equation provided that it shall not exceed the actual width of the slab:

\[
b_{ef} = k x (1 - x/t_{ef}) + a
\]

Where:
- \( b_{ef} \) = Effective width of slab,
- \( k \) = Constant having the values given in Table 8.1 depending upon the ratio of the width of the slab (\( t \)) to the effective span \( t_{ef} \),
- \( x \) = Distance of the centroid of the concentrated load from nearer support,
- \( t_{ef} \) = Effective span, and
- \( a \) = Width of the contact area of the concentrated load from nearer support measured parallel to the supported edge.

And provided further that in case of a load near the unsupported edge of a slab, the effective width shall not exceed the above value nor half the above value plus the distance of the load from the unsupported edge.

b) For two or more concentrated loads placed in a line in the direction of the span, the bending moment per metre width of slab shall be calculated separately for each load according to its appropriate effective width of slab calculated as in (a) above an added together for design calculations.
c) For two or more loads not in a line in the direction of the span, if the effective width of slab for one load does not overlap the effective width of slab for another load, both calculated as in (a) above, then the slab for each load can be designed separately. If the effective width of slab for one load overlaps the effective width of slab for an adjacent load, the overlapping portion of the slab shall be designed for the combined effect of the two loads.

Table 7.1 Values of $k$ for Simply Supported and continuous slabs

<table>
<thead>
<tr>
<th>$l/l_{ef}$</th>
<th>$k$ for simply supported slabs</th>
<th>$k$ for continuous slabs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>0.3</td>
<td>1.16</td>
<td>1.16</td>
</tr>
<tr>
<td>0.4</td>
<td>1.48</td>
<td>1.44</td>
</tr>
<tr>
<td>0.5</td>
<td>1.72</td>
<td>1.68</td>
</tr>
<tr>
<td>0.6</td>
<td>1.96</td>
<td>1.84</td>
</tr>
<tr>
<td>0.7</td>
<td>2.12</td>
<td>1.96</td>
</tr>
<tr>
<td>0.8</td>
<td>2.24</td>
<td>2.08</td>
</tr>
<tr>
<td>0.9</td>
<td>2.36</td>
<td>2.16</td>
</tr>
<tr>
<td>1.0 and above</td>
<td>2.48</td>
<td>2.24</td>
</tr>
</tbody>
</table>

For cantilever solid slabs, the effective width shall be calculated in accordance with the following equation:

$$b_{ef} = 1.2 \times a + a$$

Where

- $b_{ef}$ = effective width,
- $a_i$ = distance of the concentrated load from the face of the cantilever support, and
- $a$ = width of contact area of the concentrated load measured parallel to the supporting edge.

Provided that the effective width of the cantilever slab shall not exceed one-third the length of the cantilever slab measured parallel to the fixed edge.

And provided further that when the concentrated load is placed near the extreme ends of the length of cantilever slab in the direction parallel to the fixed edge, the effective width shall not exceed the above value, nor shall it exceed half the above value plus the distance of the concentrated load from the extreme end measured in the direction parallel to the fixed edge.

7.3.2.2 For slabs other than solid slabs, the effective width shall depend on the ratio of the transverse and longitudinal flexural rigidities of the slab. Where this ratio is one, that is, where the transverse and longitudinal flexural rigidities are approximately equal, the value of effective width as found for solid slabs may be used. But as the ratio decreases, proportionately smaller value shall be taken.

7.3.2.3 Any other recognized method of analysis for cases of slabs covered by 8.3.2.1 and 8.3.2.2 and for all other cases of slabs may be used with the approval of the engineer – in – charge.

7.3.2.4 The critical section for checking shear shall be as given in Clause 34.2.4.1 of IS: 456/2000

7.4 Slabs Spanning in Two Directions at Right Angles

The slabs spanning in two directions at right angles and carrying uniformly distributed load may be designed by any acceptable theory or by using coefficients given in Annex D/IS456:2000. For determining bending moments in slabs spanning in two directions at right angles and carrying concentrated load, any accepted method approved by the engineer-in-charge may be adopted.

Note: The most commonly used elastic methods are based on Pigeaud’s or wester guard’s theory and the most commonly used limit state of collapse method is based on Johansen’s yield line theory.

7.4.1 Restrained Slab with Unequal Conditions at Adjacent Panels

In some cases the support moments calculated from Table 3 for adjacent panels may differ significantly. The following procedure may be adopted to adjust them:

Calculate the sum of moments at midspan and supports (neglecting signs).
Treat the values from Table 3 as fixed end moments.
According to the relative stiffness of adjacent spans, distribute the fixed end moments across the supports, giving new support moments.
Adjust midspan moment such that, when added to the support moments from (c) (neglecting signs), the total should be equal to that from (a). If the resulting support moments are significantly greater than
the value from Table 3, the tension steel over the supports will need to be extended further. The procedure should be as follows:

Take the span moment as parabolic between supports: its maximum value is as found from (d).

Determine the points of contraflexure of the new support moments [from (c)] with the span moment [from (1)]

Extend half the support tension steel at each and to at least an effective depth or 12 bar diameters beyond the nearest point of contraflexure.

Extend the full area of the support tension steel at each end to half the distance from (3)

7.5 Loads on Supporting Beams

The loads on beams supporting solid slabs spanning in two directions at right angles and supporting uniformly distributed loads, may be assumed to be in accordance with Fig. 7.

7.6 Equivalent U.D.L

The load on beam from two way slab can be calculated as equivalent uniformly distributed load.

Short span: Triangular load.

For bending moment

\[
\text{BM} = \frac{\text{Weq} L^2}{8} = \frac{1}{2} \left( \frac{q x}{2} \right) \frac{L x}{2} \frac{L^2}{2} = \frac{L x^3}{24}
\]

\[
\text{Weq} = q \frac{L^2}{3}
\]

- q is the area load in kN/m²

For Shear
GENERAL DESIGN PRINCIPLES FOR R.C.C SLABS & BEAMS

16.1 Effective span

Effective span of a simply supported beam or slab shall be taken as clear span + the effective depth of slab or beam (or) c/c of supports whichever is less. Effective depth is from top of concrete to the centre of tensile steel.

16.3 Thickness of Slabs

S.S spanning in one direction 1/30 of span
S.S spanning in two direction 1/35 of span (shorter)
Continuous spanning in one direction 1/35 of span
Continuous spanning in two direction 1/40 of span (shorter)
Cantilever 1/12 of span
This will obviate deflection.

16.4 Bearing on walls

Solid slabs 100 mm.
Lintels: equal to depth of lintels, with 150 mm minimum.
Beams: 200 mm for spans up to 3.5 m, 300 mm for spans up to 5.5m and 400 mm for spans up to 7.0 m.

The breadth of a beam shall normally be 2/3 to 1/2 of the depth, but not less than 1/3 of the depth. Good rule is 3/5th of the depth of beam.
16.5 Reduction factor for span/breadth ratio

230 mm width (or) breadth
Depth can be 380mm
Where the span/breadth ratio exceeds 30, and L is the free span between lateral restraints, stress in concrete & compression reinforcement shall be reduced by a factor (1.75 – L/40b).

16.7 Reinforcement

Minimum tensile reinforcement in beams shall not be less than 0.003% where plain bars are used and 0.002% where high – yield strength deformed bars are used of the gross C.S area of the beam. Maximum area shall not exceed0.04bD. At least ¼ should be taken straight into the support.

16.8 Spacing of reinforcement bars

The horizontal clear space between two parallel main reinforcement bars shall be not less than the greater of the following.
(a) θ of bars if θ are equal
(b) The θ of the larger bar if the θ are unequal
(c) 5mm more than the nominal maximum size of the coarse aggregate used in the concrete.

The clear vertical space between the two horizontal main reinforcing bars shall normally be 15mm, the maximum size of the coarse aggregate or the maximum size of the bar, whichever is the largest.

16.9 Notes on design of Floor & Roof Slabs

The overall thickness of a slab shall be not less than 75 mm.
Minimum reinforcement: 0.15 % for M.S
0.12 % for H.Y.S.D bars
The spacing of main tensile bars shall be not more than three times, and the distributing bars not more than five times the effective depth (or) 450 mm whichever is less.
The diameter of the main tensile reinforcements in slabs shall not exceed 1/8 of the total thickness of slab and be not less than 6 mm in θ; and bars of θ > 18 mm shall not be used. Check for bond & shear if superimposed load exceeds 2kN/m².
Binding wire 16 gauge
4.5 kg of binding wire per tonne of reinforcement bar is required for tying.
Stiffness factor = M.I / length = I/l

16.10 Shear

\[ V_u = \text{Net shear force} \]
\[ V_a = \text{shear capacity of concrete} = \tau_c bd \]
\[ \tau_c = \text{permissible shear stress based on % of reinforcement.} \]
\[ b = \text{width of the section} \]
\[ d = \text{effective depth of the section.} \]

Case 1 : For \[ V_u \leq 0.5 V_a \]
No shear reinforcement is really needed.
Case 2: For $0.5 \leq V_a \leq V_u < V_c$

Provide only nominal transverse reinforcement as given below

\[ s_v \leq A_{sv} \frac{f_y}{(0.4b_w)} \]

\( A_{sv} \) = area of cross section of stirrups (area of all the legs)

\( f_y \) = yield strength of steel used for stirrups.

\( b_w \) = width of web; Spacing < 0.75d and 450mm

Case 3: For $V_a \leq V_u \leq V_{c,\text{max}}$

\( V_{c,\text{max}} = \) maximum permissible shear stress.

(Eg: For M20, \( V_{c,\text{max}} = 1.8 \) MPa)

The transverse reinforcement be so designed that the tensile stress in excess of that allowable on concrete is resisted by the reinforcement.

\[ s_v \leq \left( A_{sv} \sigma_{sv} d \right) / (V - V_a) = \left( A_{sv} \sigma_{sv} \right) / (b(\eta - \eta_v)) \]

Subject to the limit as in case 2

Case 4: For $V_u \geq V_{c,\text{max}}$

Redesign the size of web such that $V < V_{c,\text{max}}$

Notes: The stirrups bent at 90° around a bar

Table 16.1 Minimum development length of stirrups beyond the curves in mm

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>Minimum development length of stirrups beyond the curves in mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>65</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
</tr>
</tbody>
</table>

Shear Stresses and Stirrups

The intensity of shear stress $S$ (nominal shear stress) at any C.S in beams or slabs of uniform depths = $St / bd$

\( St = \) Total shear force across the section (design loads)

\( b = \) breath of the beam

\( d = \) effective depth of the beam

Shear reinforcement shall be provided to carry a shear equal to

\( St - (Sp x bd) = SR \)

\( Sp = \) Permissible shear stress as in IS 456

Shear reinforcement may be provided either

(a) By bent-up bars.

At least one-quarter of the total tensile reinforcement must be carried straight beyond the face of the supports to provide adequate anchorage.

The angle of the bend ($\varphi$) is about 30° in shallow beams with $d < 1.5b$ and 45° in other beams.

Shear resistance of bent bars is $SR = Aw x fs \sin \varphi$.

When bent up bars are provided, their contribution towards shear resistance shall not be more than half that of the total shear reinforcement.

(b) By Vertical Stirrups

Shear force to be resisted by stirrups
16.10.2 Spacing of shear reinforcement

The maximum spacing of shear reinforcement measured along the axis of the member shall not exceed 0.75d for vertical stirrups and d for inclined stirrups @ 45°, where d is the effective depth of the section under consideration. In no case shall the spacing exceed 300 mm.

Stirrups shall not be spaced further apart than a distance equal to the lever arm jd, and should preferably be spaced closer, say, at a distance equal to or less than ¼d and with a minimum spacing of ¼d at the ends which need not be less than 10cms from practical considerations.

When compression reinforcements is provided, the stirrups shall not be spaced further apart than 12times the diameter of these bars. Maximum θ of stirrups shall not be more than (d / 50).

Shear force resisted by inclined stirrups or a series of bars bent-up @ different cross-sections.

\[ SR = \frac{Awfsd}{p} \quad \text{or} \quad p = \frac{Awfsd}{SR} \]

Where,
- Aw = Total C.S area of stirrups legs (or) bent up bars within a distance of p.
- fs = Permissible tensile stress for shear reinforcement
- p = Pitch (or) spacing of the stirrups (or) bent up bars along the length of the member
- θ = angle between the inclined stirrups or bent up bars and the axis of the member
- d = effective depth.
- SR = Strength of shear reinforcement to be provided for.

16.11 Bond and Anchorage

\[ \frac{\pi d^2}{4} x fs = \pi dL f_B \]

Where,
- L = ((fsd) / 4f_B)
- f_B = ((Asfs) / L x O)
- d = dia of bar
- fs = actual tensile stress in steel
- L = development length of bar to be embedded
- f_B = permissible average bond stress for anchorage
- O = perimeter of the bar

Minimum length of embedment = fsd / 4f_B

16.11.1 Bond length for compression reinforcement

\[ L = \frac{fcd}{5f_B} \] with minimum of 24d,
fc = actual compression stress in concrete

16.12 Laps in bars

The length of the overlapping (or splicing) in joints of bars is the same as the bars lengths.

(a) For bars in flexural tension
\[ L = \frac{fsd}{4f_B} \] (or) 30d whichever is greater.

(b) For bars in direct tension
\[ 2L \] (or) 30d whichever is greater.
The Straight length of lap shall not be less then 15d (or) 20cms.

(c) For bars in compression
\[ L = \frac{fcd}{5f_B} \] (or) 24d whichever is greater.
CHAPTER 8 BEAMS (CLAUSE 23 OF IS:456/2000)

8.1 Effective depth

Effective depth of a beam is the distance between the Centroid of the area of tension reinforcement and the maximum compression fibre of laid concrete. This will not apply to deep beams.

8.2 Control of deflection (Clause 23.2 of IS: 456/2000)

The deflection of a structure or part thereof shall not adversely affect the appearance or efficiency of the structure or finishes or partitions. The deflection shall generally be limited to the following:

a) The final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from the as-cast level of the supports of floors, roofs and all other horizontal members, should not normally exceed span / 250.

b) The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes should not normally exceed span / 350 or 20 mm whichever is less.

7.2.1 The vertical deflection limits may generally be assumed to be satisfied provided that the span to depth ratios is not greater than the values obtained as below.

a) Basic values of span to effective depth ratios for spans up to 10 m;
   Cantilever
b) For spans above 10 m, the values in (a) may be multiplied by 10 / span in metres, except for cantilever in which case deflection calculations should be made.

c) Depending on the area and the stress of steel for tension reinforcement, the values in (a) or (b) shall be modified by multiplying with the modification factor obtained as per Fig. 4.

d) Depending on the area of compression reinforcement, the value of span to depth ratio is further modified by multiplying with the modification factor obtained as per Fig. 5.

\[ A_s = 0.03 \frac{A_y}{A_y} \]

\[ A_y = \text{Area of cross-section of steel required} \]
\[ A_y = \text{Area of cross-section of steel provided} \]

**Fig-8.1** Modification factor for Tension reinforcement

**Fig-8.1** Modification factor for Compression reinforcement

**8.3 Slenderness limits for beams to Ensure Lateral Stability**

A simply supported or continuous beam shall be so proportioned that the clear distance between the lateral restraints does not exceed 60 b or 250 b^2 / d whichever is less, where d is the effective depth of the beam and b the breadth of the compression face midway between the lateral restraints.

For a cantilever, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed 25 b or 100 b^2 / d whichever is less.

The breadth of a beam shall normally be 2/3 to 1/3 of the depth, but not less than 1/3 of the depth. Good value is 3/5 th of the depth of beam.

**Example:** -
Width (or) breadth = 230 mm
Depth can be = 380 mm
Where the span / breadth ratio exceeds 30, and L is the free span between lateral restraints, stress in concrete and compression reinforcement shall be reduced by a factor (1.75- L / 40B)

8.4 Design steps for Singly reinforced rectangular beams

Design procedure for design of beams by limit state method: (IS 456:2000)

8.4.1.1 A trial section is assumed (span/depth ratio & depth- breadth ratio) and self weight is calculated. (Refer clause 23.2.1)
8.4.1.2 Find factored load
8.4.1.3 Calculate effective span (Refer clause 22.2)
8.4.1.4 Calculate design B.M. & S.F.
8.4.1.5 Find effective depth (d) required (for strength) based on \[ \frac{\mu_{Limit}}{bd^2} \]
(Refer Annex-G)
8.4.1.6 Round off this value D and calculate d which shall be slightly more than \( d_{req} \).
8.4.1.7 Modify self weight as per this depth and repeat steps 7.4.1.2, 7.4.1.4, 7.4.1.5.
8.4.1.8 Find minimum reinforcement: \( \frac{A_{st}}{bd} \) (Refer Annex-G).
8.4.1.9 Alternatively take (\( \frac{M_u}{bd^2} \)) from Table D of IS 456 design aid (SP-16) and find d.
8.4.1.10 Keep the effective depth slightly more than “d” found for designing the section as a singly reinforced section.
8.4.1.11 Then find the value of (\( \frac{M_u}{bd^2} \)) and get the value of \( P_t \) from relevant table in SP-16 to calculate \( A_{st} \).

No. of bars / 1.2 M = 6
No. of bars / M = 6/12 = 5
\( A_{st} = 5 \times 113 = 565 \text{ mm}^2 \)
\( \% A_{st} = \frac{565 \times 100}{1000 \times 154} = 0.37 \)
Permissible – 0.36 + (0.12/0.25) \times 0.12 = 0.42 > 0.21
Hence satisfactory.

Check for stiffness:

Basic \( \frac{l}{D} = 20 \% \) reinforcement 0.37%

Modification factor
0.58 fy = 0.58 \times 415 = 240.7

Modification factor 1.38 (from graph)

Effective depth required for stiffness = 3500/(20 x 1.38) = 127 mm< 154 mm
Hence satisfactory
8.4.1.12 The characteristic loads are calculated and the design loads are computed by multiplying the characteristic load by appropriate partial safety factors.

8.4.1.13 Suitable diameter, Number/spacing of bars may be provided satisfying the minimum requirement and the maximum permitted values.

8.4.1.14 Minimum $A_{st} = \frac{0.85bd}{f_y}$ (clause 26.5.1.1 (a))

8.4.1.15 For Maximum compression reinforcement $A_{sc}$ (Refer Clause 26.5.1.2)

8.4.1.16 The flexural member has also to be designed for limit state of collapse in shear and checked for limit state of serviceability for deflection, width of crack etc.

8.4.1.17 Minimum shear reinforcement

$$\left( \frac{A_{sv}}{s_y} \right)_{\text{min}} = \frac{0.4b}{f_y}$$  (Clause 2.6.5.1.6)

8.4.1.18 Maximum spacing of stirrups shall be < 0.75d and < 300mm (clause 26.5.1.5)

8.5 Design data

Beam number @ elevation + m
Size of beam in mm
Breadth $b =$ mm
Over all depth $D =$ mm
Clear cover $c =$ 25 mm  (IS 456:2000–Clause 26.4.2)
Diameter of reinforcement proposed $f =$ mm
Effective depth $d =$ mm (For clear cover of 25mm)
Characteristic compressive strength of concrete $f_{ck}$ in MPa =  (Minimum of RCC = M20)
Characteristic yield strength of steel $f_y$ in MPa =
Factored moment (hagging/sagging) $M_u$ in kN m
Shear force due to factored loads $V_u$ in kN =

8.6 Maximum depth of neutral axis in limit state design $X_{u \text{ max}}$ in mm

(IS 456:2000- Clause 38.1)

If $f_y = 250$  $X_{u \text{ max}} = 0.53 d$
If $f_y = 415$  $X_{u \text{ max}} = 0.48 d$
If $f_y = 500$  $X_{u \text{ max}} = 0.46 d$

8.7 Mu Limit

$M_u = 0.36 f_{ck} b X_{u \text{ max}} / 1000 \times 1000$ (Minimum of RCC = M20) kN m  (IS 456:2000)

If $M_u < M_{u \text{ lim}}$
The beam is designed as a singly reinforced beam (under reinforced)

8.8 Percentage steel Pt limit (SP: 16 Table C page number 10)

If $f_y = 250$  $P_{t \text{ lim}} = 21.97 \times f_{ck}/f_y$
If $f_y = 415$  $P_{t \text{ lim}} = 19.82 \times f_{ck}/f_y$
If $f_y = 500$  $P_{t \text{ lim}} = 18.87 \times f_{ck}/f_y$

Find Pt from,

$$\frac{0.87 f_y x P_{t \text{ lim}}}{100} x (1 - \frac{f_y}{f_{ck}} x \frac{P_{t \text{ lim}}}{100}) b d^2 \text{ Nmm}$$
Or find $A_{st} = \frac{0.5f_{ck}}{f_y} \times \left(1 - \frac{(1-4.6)xM_c}{f_{ck}xb^2d^2}\right)$ sq.mm

If $P_t > P_{t\text{im}}$, $Pt = 100 A_{st}/bd$

If percentage steel exceeds $Pt_{\text{im}}$. Increase the depth of section and redesign.

If equation 1 is used and $P_t$ obtained Area of steel in tension $A_{st} = Pt.b.d/100$ sq.mm.

Diameter of tension reinforcement proposed $f_t$ mm.

Area of $f_t = \frac{p f_t^2}{4} \text{sq mm} = A_t$

Number of bars required = $A_{st}/A_t = N_t$ (rounded off to next higher value)
Provide $f_t$ mm dia $N_t$ numbers.

Area of tension reinforcement provided $A_{stp} = N_t x A_t$ sq mm

8.9 Design shear strength of concrete (IS:456:2000-Clause 40 and Table 20)

Shear stress $\tau_v$ in MPa = $V_u \times 1000/bd$

If $f_{ck} = 20$ MPa $\tau_{c\text{ max}} = 2.8$ MPa
If $f_{ck} = 25$ MPa $\tau_{c\text{ max}} = 1$ MPa
If $f_{ck} = 30$ MPa $\tau_{c\text{ max}} = 3.5$ MPa
If $f_{ck} = 35$ MPa $\tau_{c\text{ max}} = 3.7$ MPa
If $f_{ck} = 40$ MPa $\tau_{c\text{ max}} = 4.0$ MPa

If $\tau_v > \tau_{c\text{ max}}$ (actual shear stress exceeds the maximum shear stress) size of section will be increased and redesigned.

$P_{t_{bd}} = \frac{100A_{st}}{bd}$

Design shear strength of concrete $\tau_c$ is as in Table 19 of IS: 456:2000.
Shear capacity of concrete section

$V_c = \frac{\tau_{c}\text{bd}}{1000}$ kN

If $V_c > V_u$ provide minimum shear reinforcement. Otherwise, shear reinforcement is to be provided.
Shear to be carried by stirrups = $V_{us}$ kN.

$V_{us} = (V_u = V_c)$ kN (IS:456:2000 Clause 40.4)

Diameter of bar proposed to be used for stirrups $f_2$ mm

Number of legs $n_e$

Area of vertical legs $A_{sv} = n_e \times \frac{p f_2^2}{4} \text{sq mm}$

Spacing of stirrups $S_v$ in mm (IS 456:2000 Clause 40.4)

$S_v = 0.87 f_y A_{sv} \times d/V_{us} \times 1000.$

Check whether the spacing of stirrups $S_v$ less than or equal to lesser of the following.

(a) $S_v = 0.75 d$ IS 456:2000 Clause 26.5.1.5
(b) $S_v = 300$ mm
(c) $S_v = A_{st} \times 0.87 f_y/0.4 b$

$S_v$ is the lowest of all the above. Provide $f_2$ mm diameter $n_e$ legged stirrups @ $S_v$ mm spacing.

8.10 Development length and anchorage

Stress in steel $ss = ss$ MPa (SP:16 5.1 of Page 183)
$ss = 0.87 f_y$ MPa

for developing full strength in the bar

if $f_{ck} = 20$ $\tau_{bd} = 1.2 \times 1.6 = 1.92$
if $f_{ck} = 25$ $\tau_{bd} = 1.4 \times 1.6 = 2.24$
if $f_{ck} = 30$ $\tau_{bd} = 1.5 \times 1.6 = 2.40$
if $f_{ck} = 35$ $\tau_{bd} = 1.7 \times 1.6 = 2.72$
if $f_{ck} = 40$ $\tau_{bd} = 1.9 \times 1.6 = 3.04$

$L_{vd} = f_t \times s/4 \tau_{bd} = \text{mm}$ (IS 456:2000-Clause 26.2.1)

Development length of bar $L_{vd}$ mm
Abstract

Size of beam

Breadth \( b \) mm =
Overall depth \( D \) mm =

Reinforcement

Reinforcement to take up tension \( f_t \) mm
Dia \( N_t \) nos.
Stirrups \( N_s \) legged \( f_s \) dia @ a spacing of \( S_v \) mm
Development length of bar = \( L_d \) mm

8.11 Design steps for doubly reinforced rectangular beams

8.11.1 Why doubly reinforced sections?

Compression steel is required for the following reasons
i) To increase the ultimate M.R of section with restricted C.S dimensions
ii) To increase the rotation carrying capacity of section
iii) To increase the stiffness of section
iv) To account for the reversal of moment.

8.11.2 Design steps :

In the case of doubly reinforced beams, the size of the beam is already known. The design involves calculation of reinforcement required.

- Design load including self weight is to be determined.
- Find effective span similar to S.S beam
- Calculate design B.M. (\( M_u \)).
- \( M_u \text{, limit} = Qbd^2 \) ---- Find the MR of singly reinforced beam.
- If \( M_u < M_u \text{, limit} \), it is designed as a singly reinforced beam.
- If \( M_u > M_u \text{, limit} \), it is to be designed as a doubly reinforced section.
- The extra BM = (\( M_u - M_u \text{, limit} \)) = \( M_u2 \)
- Value of maximum NA, \( X_u \text{, Max} \) is taken from code.
- Area of compression reinforcement = \( A_{sc} \frac{M_u2}{(f_{sc} - f_{cc})(d - d') \text{, max}} \)

Where \( f_{sc} \) = stress in compression reinforcement corresponding to a strain of 0.0035 \( \frac{M_u \text{, max} - d^*}{M_u \text{, max}} \)

\( f_{cc} \) = compression stress in concrete at the level of centroids of compression reinforcement.

\( d^* \) = effective cover to compression reinforcement.

Approximate area to compression reinforcement (appendix E-1.2-IS: 450)

\( A_{sc} = \frac{M_u2}{(f_{sc})(d - d') \text{, max}} \)

- Area of tension steel required (\( A_{st1} \)) to develop the limiting moment of resistance (\( M_u \text{, limit} \)) is determined as in the case of singly reinforced balanced sections (or) from the % tension steel (\( p_{t \text{, max}} \)) of balanced section.
- Area of tension steel required (\( A_{st2} \)) for \( M_u2 \).
The area of tension steel required to develop the additional M.R can be determined also as a beam.

\[ A_{st2} = \frac{M_{u2}}{0.87f_y} \]

\[ A_{st2} = \frac{A_{sc}x_{sc}}{0.87(f_y)(d-d''')} \]

- Total area of tension steel = \( A_{st} = A_{st1} + A_{st2} \)

- Abstract :-
  
  (a) Breadth b mm
  (b) Over all depth D mm
  (c) Reinforcement to take up tension \( f_t \) mm dia \( N_t \) nos
  (d) Reinforcement to take up compression \( f_c \) mm dia \( N_c \) nos
  (e) Stirrups \( N_e \) legged f dia @ a spacing of \( S_e \) mm
  (f) Development length of bar Ld mm.

8.11.3 Design Data

Beam Number @ Elevation + ............. m
Size of beam in mm
Breadth b = ........ mm
Depth D = ......... mm
Clear cover c = 25 mm (IS 456:2000–Clause 26.4.2)
Diameter of reinforcement proposed \( f = ......... \)
Effective depth \( d = D - \frac{1}{2} - 25 = .... \)
Characteristic compressive strength of concrete \( f_{ck} \) in MPa =
Characteristic yield strength of steel \( f_y \) in MPa =
Factored moment hogging /sagging \( M_u \) in kNm =
Shear force due to factored loads \( V_u \) in kN =

8.11.4 Maximum depth of neutral axis in limit state design \( X_{u_{max}} \) in mm

(IS 456:2000- Clause 38.1)

- if \( f_y = 250 \) \[ X_{u_{max}} = 0.530 d \]
- if \( f_y = 415 \) \[ X_{u_{max}} = 0.48 d \]
- if \( f_y = 500 \) \[ X_{u_{max}} = 0.46 d \]

8.11.5 Mu Limit

\( M_u = \{0.36 f_y b X_{u_{max}} (d-0.416 X_{u_{max}})/1000 \times 1000\} \) = kNm (IS:456:2000)
if \( M_u > M_{u_{lim}} \) the beam is designed as a doubly reinforced beam.
If \( M_u < M_{u_{lim}} \) the beam is designed as a singly reinforced beam.

8.11.6 Percentage steel \( P_{t_{lim}} \) (SP:16 Table C page number 10)

- If \( f_y = 250 \) \[ P_{t_{lim}} = 21.87 f_y/kf_y \]
- If \( f_y = 415 \) \[ P_{t_{lim}} = 19.82 f_y/kf_y \]
If $f_y = 500$  

$P_{lim} = 18.87 \times f_y/f_y$  

Area of steel in tension $A_{st} = P_{lim} \times d/100$ sq mm  

Additional moment $M_{u} = (M_u - M_{u,lim})$ kN.m  

Diameter of compression reinforcement proposed $f_c$ mm  

$D' = f_c/2 + c$  

Additional tension reinforcement $A_{st2}$ in sq mm  

$A_{st2} = \frac{M_u - M_{u,lim}}{f_y} \times d/d'$  

Additional tension reinforcement $A_{st2}$ in sq mm  

$A_{st2} = \frac{M_u - M_{u,lim}}{f_y} \times d/d'$  

(IS 456:2000-Annex G)  

If $d'/d > 0.2$ is greater than 0.2 increase the depth of beam and redesign.  

Compressive stress in concrete at the level of centroid of compression reinforcement $f_{cc} = 0.446 f_c k$ MPa  

(for simplification)  

8.11.7 Stress in compression reinforcement $f_{sc}$ MPa  

If $f_y = 250$MPa $f_{sc} = 0.87 f_y$  

If $f_y = 415$MPa and $d'/d = 0.05$ $f_{sc} = 355$ MPa  

If $f_y = 415$MPa and $d'/d = 0.10$ $f_{sc} = 353$ MPa  

If $f_y = 415$MPa and $d'/d = 0.15$ $f_{sc} = 342$ MPa  

If $f_y = 500$MPa and $d'/d = 0.05$ $f_{sc} = 424$ MPa  

If $f_y = 500$MPa and $d'/d = 0.10$ $f_{sc} = 412$ MPa  

If $f_y = 500$MPa and $d'/d = 0.15$ $f_{sc} = 395$ MPa  

If $f_y = 500$MPa and $d'/d = 0.20$ $f_{sc} = 370$ MPa  

8.11.8 Reinforcement  

Required area of compression reinforcement $A_{sc}$ in sq mm  

$A_{sc} = A_{st2} x 0.87 f_y/(f_{cc} - f_{sc})$ sq mm  

Required area of tension reinforcement $= A_{stt} = (A_{st1} + A_{st2})$ sq mm  

Diameter of reinforcement proposed for tension $f_t$ mm  

Area of bar $A_{rt} = \pi f_t^2/4$ sq mm  

No of bars required for tension $N_{t2} = A_{stt}/A_{rt}$ (Round off to next higher value)  

Area of steel provided for tension $=(N_{t2} x A_{rt})$ sq mm  

Diameter of reinforcement proposed for compression $f_{tc}$ mm  

Area of bar $A_{rtc} = \pi f_{tc}^2/4$ sq mm  

No of bars required for compression $N_{c} = A_{scp}/A_{rtc}$ (rounded off to next higher value)  

Area of steel provided for compression $= A_{scp} = (N_{c} x A_{rtc})$ sq mm  

Total area of steel provided $A_{stp} = (A_{stt} + A_{scp})$ sq mm  

$P_{sp} =$ percentage steel provided $= (A_{stp}/bd \times 100)$  

8.11.9 Design for shear(IS:456:2000-Clause 40 and Table 20)  

Shear stress $\tau_c$ in MPa (SP:16 cl.4.2) $= V_u \times 1000/b_w \times d$  

if $f_{tk} = 20$ MPa $\tau_{c,max} = 2.8$ MPa  

if $f_{tk} = 25$ MPa $\tau_{c,max} = 3.1$ MPa  

if $f_{tk} = 30$ MPa $\tau_{c,max} = 3.5$ MPa  

if $f_{tk} = 35$ MPa $\tau_{c,max} = 3.7$ MPa  

if $f_{tk} = 40$ MPa $\tau_{c,max} = 4.0$ MPa  

If $\tau_c > \gamma_{max}$ (if actual shear stress exceeds the maximum shear stress ) size of section will be increased and redesigned.  

Design shear strength of concrete $\tau_c$ as in Table 19 of IS:456:2000.  

Shear capacity of concrete section $V_c = (\tau_c b_w /d/1000)$ kN  

If $V_c > V_u$ provide nominal shear reinforcement otherwise shear reinforcement is to be provided.  

Shear to be carried by stirrups $= V_{us}$ kN $= V_{us} = (V_u - V_c)$ kN  

Diameter of bar proposed to be used for stirrups $f_{tc}$ mm  

No .of legs $N_e$  

Area of vertical legs $= A_{sv} = N_e \times p \times d/2/4$ sq mm  

Spacing of stirrups $S_v$ in mm (IS 456:2000 Clause 40.4) $= 0.87 f_y A_{sv} /d/V_u \times 1000$. Check whether the spacing of stirrups $S_v$ less than or equal to lesser of the following.  

(d) $S_v = 0.75 d$  

(IS 456:2000 Clause 26.5.1.5)
8.11 Development length and anchorage

Stress in steel = \( \sigma_s \) MPa (SP: 16 5.1 of Page 183)

\[ \sigma_s = 0.87 \, f_y \, \text{MPa} \]

for developing full strength in the bar

if \( f_{ck} = 20 \)
   \( \tau_{bd} = 1.2 \times 1.6 = 1.92 \)
if \( f_{ck} = 25 \)
   \( \tau_{bd} = 1.4 \times 1.6 = 2.24 \)
if \( f_{ck} = 30 \)
   \( \tau_{bd} = 1.5 \times 1.6 = 2.40 \)
if \( f_{ck} = 35 \)
   \( \tau_{bd} = 1.7 \times 1.6 = 2.72 \)
if \( f_{ck} = 40 \)
   \( \tau_{bd} = 1.9 \times 1.6 = 3.04 \)

\[ L_d = f_t \times s_s / 4 \tau_{bd} = \text{mm} \quad \text{(IS 456:2000-Clause 26.2.1)} \]

8.12 Reinforcement

Reinforcement to take up tension \( f_{te} \) mm dia at a spacing of \( S_t \) mm.
Reinforcement to take up Compression \( f_{ce} \) mm dia at a spacing of \( S_c \) mm.
Development length of bar \( L_d \) mm.

8.13 Cantilever beam

- Atleast 50% of the tension reinforcement provided at the support should extend to the end of the cantilever. The remaining 50% should extend a distance of 0.5L or 45 times the bar size which ever is greater from the support.
- (Span/depth) ratio is 5 to 7 for cantilever slab/beam.
- Width of the cantilever beam should not be less than \( L/25 \) or \( \sqrt{L \times d \times 10} \) whichever is greater, where \( L \) is the clear distance between the lateral restraint and \( d \) is the effective depth. Generally the width of the beam is kept 1/3 to 2/3 of its depth.
- Design procedure for design of beams of limit state (Collapse) of beams
  1) A trial section is assumed for calculating the self weight.
  2) The characteristic loads calculated and the design loads are computed by multiplying the characteristic load by appropriate partial safety factors.
  3) The effective span of the beam is determined.
  4) Calculate the design B.M and S.F.
  5) By equating the limiting moment of resistance (M.R. of balanced section) to design B.M, find the effective depth (d) required based on strength. From this calculate overall depth (D)
      \[ D = d + \text{clear cover} + \frac{1}{2} \text{dia of the bar proposed} \]
  6) The area of the tension steel required may be determined by two methods
     a) If the effective depth provided is equal to the effective depth required, the section is a balanced one and hence the percentage tension steel required \( (p_{t, \text{max}}) \) may be obtained from the table below and the area of steel calculated

8.11 Abstract

breadth ‘b’ mm =
overall depth ‘D’ mm =
clear cover 25 mm =
<table>
<thead>
<tr>
<th>GRADE OF CONCRETE</th>
<th>$P_{t \text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe 415</td>
<td>Fe 500</td>
</tr>
<tr>
<td>M20</td>
<td>0.955</td>
</tr>
<tr>
<td>M25</td>
<td>1.194</td>
</tr>
</tbody>
</table>

(b) The exact area of steel required for the under reinforced sections (when depth provided is more than that required) can be determined using

$$M_u = 0.87f_y A_{st} \left[ d - \left( \frac{f_y}{f_{ck}} \times b \right) \right]$$

7) Suitable dia, number / spacing of bars may be provided satisfying the minimum requirements and the maximum permitted values.

$$A_{st \text{ min}} = 0.85 \frac{bd}{f_y}$$

$$A_{st \text{ max}} = 0.04 bd$$

- The flexural member has to be also designed for limit state of collapse in shear and checked for limit state of serviceability for deflection width of crack etc.

- Minimum shear reinforcement

$$\left( \frac{A_{sv}}{S_v} \right)_{\text{min}} = 0.4h / f_y$$

- Maximum spacing

$$d \geq 0.75d$$

$$d \geq 300\text{mm}$$

**Example 8.1:**

_Calculate the ultimate moment carrying capacity of a R.C beam section $b = 250\text{mm}$; $D = 500\text{mm}$; $A_{st} = 3$ numbers $25\text{mm}$ dia; $f_{ck}=M20$; $f_y = 415\text{MPa}$_

$D$ is the Overall depth

$$D = 500 - 25 - 12.5 = 462.5$$

$$P_t = \frac{1472 \times 100}{250 \times 462.5} = 1.273\%$$

$$M_u = 0.87 f_y \left( \frac{P_t}{100} \right) \times \left[ 1 - 1.005 \times \frac{f_y}{f_{ck}} \left( \frac{P_t}{100} \right) \right] bd^2$$

$$= 0.87 \times 415 \times \left( \frac{1.273}{100} \right) \times \left[ 1 - 1.005 \times \frac{415}{20} \times \left( \frac{1.273}{100} \right) \right] \times 250 \times 462.5^2$$

$$= 4.6 \times (0.7345) \times 53.476 \times 10^6 = 180.7\text{kNm}$$
A rectangle beam of 300mm x 500mm over all size is reinforced by 6 numbers 25mm dia bars. Calculate the ultimate moment of the beam if M20 mix is used and $f_y = 415\, \text{MPa}$.

$$d = 500 - 25 - 12.5 = 462.5\, \text{mm}$$

$$Pt = \frac{2945 \times 100}{300 \times 462.5} = 2.12\%$$

$$= 0.87 \times 415 \times \left(\frac{2.12}{100}\right) \times \left[1 - 1.005 \times \frac{415 \times 2.12}{20 \times 100}\right] \times 300 \times 462.5^2$$

$$= 7.654 \times (0.558) \times 300 \times 462.5^2 = 274\,\text{kNm}$$

**Example 8.3:**

**Design a singly reinforced concrete section for a simply supported rectangle beam with a span of 5m to carry a dead load of 25 kN/m and imposed load of 15 kN/m. Use M20 mix and $f_y = 415\,\text{MPa}$**

**Factored load**

$$Wl = 1.5 \times (25 + 15) = 60\,\text{kN/m}$$

$$Mu = \frac{Wl^2}{8} = \frac{60 \times 25}{8} = 187.5\,\text{kNm}$$

**To Find d:**

$$\frac{Mu}{bd^2} = 2.76 (\text{Limit})$$

$$d = \sqrt[3]{\frac{Mu}{b \times 2.76}}$$

$$b = 230\,\text{mm}$$

$$= \sqrt[3]{\frac{187.5 \times 10^6}{230 \times 2.76}} = 543\,\text{mm}.$$  

**Keep D = 600**

$$d = 600 - 25 - 12.5 = 562.5\,\text{mm}$$

$$\frac{Mu}{bd^2} = \frac{187.5 \times 10^6}{230 \times 562.5^2} = 2.576$$

$$Pt = 0.874$$

$$Ast = \frac{0.874 \times 230 \times 562.5}{100} = 1131\,\text{mm}^2$$

Provide 4-20\( \phi \) (Ast = 1256mm\(^2\))

$$P_t = \frac{1256 \times 100}{230 \times 562.5} = 0.97$$

**Shear:**

$$\tau_c^* = 0.61 \,\text{MPa} \, (\text{Design Shear Strength})$$

$$\text{Shear \ Fore} = \frac{60 \times 5}{2} = 150\,\text{kN}$$
Shear Capacity of Concrete = \( \frac{0.61 \times 230 \times 562.5}{1000} \) = 78.9kN

Balance \( V_{as} = 150 - 78.9 = 71.1 \) kN

\( \frac{V_{as}}{d} = \frac{71.1}{56.25} = 1.264 \) kN/cm

Two Legged 8\( \phi \) @ 250c/c
Spacing not to exceed = \( 0.75d = 0.75 \times 562.5 = 422 \) mm
Spacing not to exceed 300mm

Checking for Minimum (Clause 26.5.1.6)

\[
\frac{A_{sv}}{bs_y} = \frac{0.4}{0.87 f_y}
\]

\[
\therefore A_{sv} = \frac{0.4 \times 230 \times 250}{0.87 \times 415} = 63.7 \text{mm}^2
\]

Provided is 100mm\(^2\)
Hence satisfactory

**Example 8.4:**

*A reinforced concrete beam of 230mm by 450mm effective depth has to resist a moment of 100kN-m. Determine the area of steel required. \( f_y = 415\text{MPa} \)

**Concrete grade M20**

\[
Mu = 0.87 f_y A_{st} \left[ d - \frac{f_y A_{st}}{f_{ck} b} \right]
\]

\[
D = 450\text{mm}
\]

\[
\therefore d = 450 - 25 - 10 = 415\text{mm}
\]

\[
b = 230\text{mm}
\]

\[
Mu = 0.87 \times 415 \times A_{st} \left[ 415 - \frac{415 \times A_{st}}{20 \times 230} \right] = 100 \times 10^6 \text{Nmm}
\]

\[
149836 A_{st} - 32.6 A_{st}^2 = 100 \times 10^6
\]

\[
A_{st} = 811\text{mm}^2
\]

Provide 3Nos - 20mm\((A_{st} = 942\text{mm}^2)\)

As per SP: 16 Table

\[
\frac{Mu}{bd^2} = \frac{100 \times 10^6}{230 \times 415^2} = 2.5245 < 2.76 \text{(Singly Reinforced)}
\]

\[
P_i = 0.848 \text{(Table2)}
\]

\[
A_{sv} = 0.848 \times 230 \times 415 = 809\text{mm}^2
\]

**Example 8.5:**

*Design a singly reinforced concrete section for a simply supported beam with a span of 5m to carry a dead load of 25 kN/m and working imposed load of 15kN/m. use M20 and \( f_y = 500 \text{ MPa} \).*
\[ P_u = 1.5(DL+LL) = 1.5(25+15) = 60 \text{ kN/m} \]

\[ M_u = \frac{(P_u \times L^2)}{8} = \frac{(60 \times 5^2)}{8} = 187.5 \text{ kN.m} \]

Using SP:16 Tables,

\[ \frac{M_u}{bd^2} = \frac{187.5 \times 10^6}{250 \times 5^2} = 2.76 \]

For Balanced Single reinforced section

\[ d = \sqrt{\frac{187.5 \times 10^6}{250 \times 2.76}} = 521 \text{ mm} \]

Provide Overall depth of 560mm

\[ d = 560 - 25 - 10 = 525 \text{ mm} \]

\[ \frac{M_u}{bd^2} = \frac{187.5 \times 10^6}{300 \times 525^2} = 2.267 \]

Let \( b = 300 \text{ mm} \)

Pt = 0.643 (Table 2)

\[ A_{st} = \frac{0.643}{100} \times 300 \times 525 = 1012 \text{ mm}^2 \]

Provide 5 Nos of 16\( \phi \) (\( A_{st} = 1005 \text{ mm}^2 \))

**Example 8.6:**

*A doubly reinforced beam of width 300mm and effective depth 500mm is reinforced as shown.*

*Calculate the moment of resistance. If \( f_y = 415 \text{ MPa} \) and M30 is used.*

**Step-1:** To find the moment resistance in concrete as singly reinforced beam.

\[ M_{uc1} = 0.138 \times f_{ck} \times bd^2 \]

\[ = 0.138 \times 30 \times 300 \times 500^2 \]

\[ = 310 \text{ kN.m} \]

**Step-2:** To find balanced steel \( A_{st1} \)

Balanced steel \( pt = 19.82 \times 30/415 \) (From Table-C SP 16)

\[ = 1.4\% \]

\[ A_{st1} = \frac{(1.43 \times 300 \times 500)}{100} = 2145 \text{ mm}^2 \]
Step-3: To find out compression stress in Asc
\[ d'/d = 50/500 = 0.1 \]
Fe 415
\[ f_{sc} = 0.1 \]
(From Table-F SP 16)
\[ = 353 \text{ N/mm}^2 \]
Compression in steel = \( f_{sc} \times A_{sc} \)
\[ = 353 \times 942 \]
\[ = 333 \text{ kN} \]

Step-4: Moment in compression due to compression steel
\[ M_{uc2} = f_{sc} \times A_{sc} \left( L \cdot A \right) \]
\[ = 333 \times (500-50) \times 10^{-3} \]
\[ = 150 \text{ kN.m} \]

Step-5: Total resisting moment
\[ M_{uc} = M_{uc1} + M_{uc2} \]
\[ = 311 + 150 \]
\[ = 461 \text{ kN.m} \]

Step-6: Resisting moment for tension failure
Additional steel available in tension
\[ A_{st2} = A_{st1} \]
\[ = 2454 - 2145 \]
\[ = 309 \text{ mm}^2 \]

Step-7: Mut limiting moment in steel
\[ M_{ut} = M_{uc1} + 0.87 f_y A_{st2} \left( L \cdot A \right) \]
\[ = 311 + 0.87 \times 415 \times 309 \times (450 \times 10^{-3}) \]
\[ = 311 + 502 \]
\[ = 813 \text{ kN.m} \]

Step-8:
Resisting moment in compression failure = 461 kN.m
Resisting moment in tension failure = 813 kN.m
Hence Resisting moment is 461 kN.m

### 8.14 Beam on elastic foundation

#### 8.14.1 Modulus of foundation

<table>
<thead>
<tr>
<th>Type of Soil</th>
<th>K (modulus of foundation) in</th>
</tr>
</thead>
<tbody>
<tr>
<td>*** Psi/in</td>
<td>Mpa/mm</td>
</tr>
<tr>
<td>Very poor sub grade</td>
<td>0 - 150</td>
</tr>
<tr>
<td>Poor sub grade</td>
<td>150 - 180</td>
</tr>
<tr>
<td>Fair to good sub grade</td>
<td>180 - 250</td>
</tr>
<tr>
<td>Excellent sub grade</td>
<td>250 - 300</td>
</tr>
<tr>
<td>Good sub grade</td>
<td>300 - 600</td>
</tr>
<tr>
<td>Good base</td>
<td>600 - 700</td>
</tr>
<tr>
<td>Best base</td>
<td>700 - 800</td>
</tr>
</tbody>
</table>
Fair to good sub grade = 180-250

\[ P = 180 \delta = 180 \text{in (Psi/in)} \]

‘\( \delta \)’ is in ‘mm’@ Mid point of a 12 inches wide (305 mm) and 18 inches (457 mm) deep beam sitting over the fair to good sub grade.

Let us say that the beam carries Brick work for a height of 10 feet (3048 mm) and the effective span = 12 feet (3658 mm).

Weight of brickwork/m = \( 1 \times 3.048 \times 0.27 \times 20 = 16.46 \text{ kN/m} \)

Self weight = \( 0.305 \times 0.457 \times 1 \times 25 = 3.4846 \text{ kN/m} \)

Total = 19.94 kN/m

\[ E = 5000 \sqrt{fck} = 5000 \sqrt{20} = 22361 \text{ N/mm}^2 = 22.361 \times 10^6 \text{ kN/Sq m} \]

\[ I = (305 \times 457^3 / 12) \times 10^{-12} = 2.425868 \times 10^{-3} \text{ m}^4 \]

\( \delta \)@ center = \( (5/384) \times (W^4 / EI) \)

= \( (5/384) \times [(19.94 \times 3.658^4) / (22.361 \times 10^6 \times 2.425868 \times 10^{-3})] \) = 0.86 mm

= 0.034 in

\[ P = 180 \times 0.034 = 6.09 \text{ Psi} \]

\[ = 4281.7 \ \text{kg/sq.m} = 42 \text{ kN/m}^2 \times 42 \times 0.305 = 12.81 \text{ kN/M} \]

This is about \( [(12.81 \times 100 / 19.94) = 64.24\%] \)

In S.I Units:-

Fair to good sub grade = 0.0488 to 0.0678

\[ P = 0.0488 \ \delta = 0.0488 \text{ mm (MPa/mm)} \]

‘\( \delta \)’ is in ‘mm’@ Mid point of a beam sitting over the fair to good sub grade.

\[ P = 0.0488 \times 0.86 = 0.0412 \text{ MPa} = 41.2 \text{ kN/Sq m} = 41.2 \times 0.305 = 12.566 \text{ kN/M} \]

This is about \( [(12.566 \times 100 / 19.94) = 63\%] \)

**8.14.2 Design of plinth beam:**

Plinth beams are provided at ground level keeping top of the beam at Natural ground level or at plinth level, (i.e.,) at finished floor level. The beams resting on ground is on elastic foundation. The beam at plinth level (FFL) which is above the natural ground will normally rest on masonry (B.W or R.R). In both the cases the beam is supported throughout. The plinth beams are provided to break the height of masonry from foundation to lintel level or to roof level (i.e.) to act as tie for columns in case of framed structures. In any case, the design of such beams are to be done “considering beams on elastic foundations”.

**Example 8.7:**

**Case I:**

Here an attempt is made in a different way as lintels are designed.

Density of brick work = 20 kN/cum.

Thickness of wall = 343mm.

Weight = \( \frac{1}{2} \times 1230 \times 8 \)

= 10.08 kN.
Maximum Bending moment = \[ \frac{wl}{6} = \frac{10.08 \times 2.4}{6} = 4.03 \text{kNm} \]

\[ M_u = 1.5 \times 4.03 = 6.45 \text{kNm} \]

Let the width be = Thickness of B.W

\[ \frac{M_u}{bd} = 27 \]

for M20 mix and FE415 MPa.

Therefore \( d = \frac{604810}{350276} = 79.11 \text{ mm} \).

Provide 230mm overall depth.
Effective depth = 230-50-8 = 172mm,

\[ \frac{100}{172} \]

Provide 2 # 12 mm \( A_{st} = 226 \text{ mm}^2 \).

Case II:

Considering that a beam is provided at ground level and a plinth beam at finished floor level, the load between these two beams will be taken up by the beam at ground level. In the earlier case we have considered triangular load. In the present case, we will consider the load between beams.

Weight of B.W. = \[ 1 \times 0.35 \times 20 \times 1.2 = 8.4 \text{ kN/m} \]

\[ M = \frac{8.4 \times 2.4^2}{10} = 4.84 \text{kNm} \]
Providing over all depth as 230mm;  

\[ M_u = 1.5 \times 4.84 = 7.26 \text{kNm}. \]

\[ \frac{M_u}{bd^2} = 2.76 \]

\[ d = \sqrt{\frac{7.26 \times 10^6}{350 \times 2.76}} = 86.7 \text{mm}. \]

Providing over all depth as 230mm;  

\[ d = 230.6 \text{mm} \]

\[ \frac{M}{bd} = \frac{7280}{350.84} \]

Using M20 mix and FE415 MPa

\[ P_t = 0.172 \]

Minimum

Provide 2 # 12 mm Φ.

**Case III:**

Plinth beam at ground level supported by columns (or) piles at 2.4m interval.

Height of Compound wall = 2.7m

Thicknenss of Compound wall = 230 mm.wt/m

\[ = 1 \times 0.23 \times 2.7 \times 20 = 12.42 \text{kN/m say } 12.5 \text{kN/m} \]

\[ M = \frac{12.5 \times 2.4^2}{10} = 7.2 \text{kNm}. \]

Considering continuous beam, this moment creates tension at top at support location and tension at bottom at mid span.

Providing 230mm wide beam M20 mix; FE415MPa.

\[ d = \sqrt{\frac{1840}{230.76}} \]

(+ve and _ve bending moment)

Let D=230mm \hspace{1cm} d = 230-40-6=184mm
\[ M_{u} = \frac{108 \times 10}{230 \times 184} = 1.38 \]
\[ p_{r} = 0.426 \]
\[ A_{s} = \frac{0.426 \times 230 \times 184}{100} = 180 \text{mm}^2 \]
\[ \min A_{s} = \frac{0.85 \times 230 \times 184}{415} = 87 \text{mm}^2 \]

**Shear Reinforcement**

**Case I:**

\[ \text{shearforce} = \frac{\text{totalload}}{2} = \frac{10.08}{2} = 5.04 \text{kN} \]
\[ V_{u} = 1.5 \times 5.04 = 7.56 \text{kN} \]
\[ p_{r} = \frac{226}{350 \times 172} \times 100 = 0.375 \]
\[ \tau'_{c} = 0.44 \text{MPa} \]

\[ \text{2 # 120 Hanger rods} \]

\[ \text{2 # 12 0} \]

\[ \text{STIRRUPS 80 2 Legged, @ 155mm c/c} \]

Capacity of concrete in shear = \[ \frac{0.44 \times 350 \times 172}{1000} = 26.5 \text{kN} > 7.56 \text{kN} \]

Hence provide minimum shear reinforcement

keeping 2 legged 8 diameter stirrups,

\[ s_{v} = \frac{100}{350 \times 1.84 \times 10^{-3}} = 155 \text{mm} \]

Provide 8 diameter 2 legged stirrups at 155mm c/c

This is not exceeding 0.75d and 300 mm

**Case II:**

\[ \text{shearforce} = \frac{1.5 \times 8.4 \times 2.4}{2} = 15.12 \text{kN} \]
\[ p_{r} = \frac{226}{350 \times 184} \times 100 = 0.35 \]
\[ \tau'_{c} = 0.415 \text{MPa} \]
Capacity of concrete in shear = $\frac{0.415 \times 350 \times 184}{1000} = 26.73kN > 15.12kN$

**Minimum shear reinforcement**

Let us provide 2 legged 8 diameter stirrups,

0.75$d$=0.75 $\times$ 184 = 138 mm
Provide 8 diameter 2 legged stirrups at 135mm c/c

Case III:

\[ \text{shearforce} = \frac{1.5 \times 12.5 \times 2.4}{2} = 22.5kN \]

\[ p_c = \frac{226}{230 \times 184} \times 100 = 0.53 \]

\[ \tau_c' = 0.48MPa \]

Capacity of concrete in shear = $\frac{0.48 \times 230 \times 184}{1000} = 20.3kN < 22.5kN$

\[ V_{us} = 22.5 - 20.3 = 2.2kN \]

\[ \frac{V_{us}}{d} = \frac{2.2}{18.4} = 0.12kN/cm \text{ Refer table 62} \]

Minimum shear reinforcement
Provide 8 diameter 2 legged stirrups at 0.75$d$ =0.75 $\times$ 184 =138mm c/c
Provide 8 diameter 2 legged stirrups at 135mm c/c
\[ \frac{V_{u}}{d} \text{ for this } = 2.69 \text{ kN/cm} > 0.12 \text{ kN/cm} \]

**Note:** Hanger rods can be less than 12 dia also. But at support (top) 2#12 dia are required. Hence they are not only hanger rods but also they take up bending tension at supports. Since there are only two rods; no rods could be cranked and taken up.

**Example 8.8:**

*Design of a beam resting on ground over column at an interval of 3.6m carries B.W. 230 mm thick and 3.2m height*

Continuous Beam:

Let us provide over all depth as 230mm

Self weight = 0.23 x 0.23 x 25 x 1 = 1.32 kN/m

Weight of B.W. = 3.2 x 0.23 x 20 = 14.72 kN/m

Total load = 16.04 kN/m say 16 kN/m

\[ M_u = \frac{1.5 \times 16 \times 3.6^2}{10} = 31.104 \text{kNm} \]

Using M20, Fe 415

Let us provide over all depth as 300mm

\[ d = 300 - 40 - 10 = 250 \text{ mm} \]

Self weight = 0.3 x 0.23 x 25 = 1.73 kN/m

Weight of B.W. = 3.2 x 0.23 x 20 = 14.72 kN/m

Total load = 16.445 kN/m

Design as beam on elastic foundation

Soil: fair to good sub grade
Therefore Modulus of foundation = 0.0488 to 0.0678 MPa/mm

\[ P = (0.0488 \times \delta) \text{MPa} \]

\[ \delta = \frac{5wj^4}{384EI} \]

\[ E = 5000\sqrt{fck} = 5000\sqrt{20} = 22.361 \times 10^6 \text{kN/m}^2 \]

\[ I = \frac{230 \times 300^3}{12} = 517.5 \times 10^6 \text{mm}^4 = 5.175 \times 10^{-4} \text{m}^4 \]

\[ \delta = \frac{5}{384} \times \frac{16.445 \times 3.6^4}{22.361 \times 10^6 \times 5.175 \times 10^{-4}} = 3.11 \text{mm} \]

\[ P = 0.0488 \times 3.11 = 0.152 \text{MPa} \]

\[ \text{soil reaction} / m = \frac{0.152 \times 1000 \times 230}{1000} = 35 \text{kN} > 16.1 \text{kN} \]

Hence the plinth beam is only a tie beam between columns/piles.

8.15 **Design of lintels**: Minimum Bearing on walls 150mm

a) When the height of wall above the lintel is greater than 0.866L, the load triangular as shown in Figure

W = Total weight of triangular portion of Brick Work
L = Effective Span of Lintel
M = (WL/6)

b) When the height of wall above the lintel is less than 0.866L and where no load is transferred on top of wall.
W = weight of wall portion ABCD.

b) When the height of wall above the lintel is less than 0.866L and where no load is transferred on top of wall.
W = weight of wall portion ABCD.

C) When the roof (or) floor transmits load on the wall within a height of 0.866L above the lintel.

Weight of rectangular portion of masonry = \( w_1 \text{kN/m} \).

Load from slab (Portion “a”) = \( w_2 \text{kN/m (u.d.l)} \)
When there is a wall above the slab, so that the total height of wall is greater than 0.866L, the weight of triangular portion of the masonry in addition to the roof load shall be considered.

**Example 8.9: Lintel cum sunshade.**

Clear width 1.5m; Concrete M20 & Fe415;
Height of wall above lintel 1.8m; Thickness of wall + plastering, 230+40=270mm;
Sunshade projection 750mm; Bearing of lintel 200mm

Imposed load on sunshade = 0.75kN/m²; (inaccesable)

Density of B.W.=20kN/cum.

**Design of sunshade:**

Cantilever 750mm;
Span/depth = 750/7 = 107mm (Clause 23.2.1 of IS 456:2000)

Let the overall depth at fixed end be 75mm and at free end be 50mm

Effective depth = 75-15-5=55mm.

Density of concrete=25 kN/cum.

Considering 1m length of sunshade.

Self weight = \(1 \times \frac{0.075 + 0.05}{2} \times 0.75 \times 25 = 1.172\) kN/m

Imposed load =\(0.75 \times 1 \times 0.75 = 0.563\) kN/m

Total load = 1.735 kN/m

Load factor 1.5

Design load=1.735 x 1.5 =2.6 kN/m width

Maximum Bending Moment

For Sunshade = \(\frac{2.6 \times 0.75^2}{2} = 0.731\) kNm

\[ \frac{Mu}{bd^2} = 2.76 \]

\[ d = \sqrt{\frac{0.731 \times 10^6}{1000 \times 2.76}} = 16.3\text{mm} \]

Provided = 75 – 15 – 5 = 55mm

\[ \frac{Mu}{bd^2} = \frac{0.731 \times 10^6}{1000 \times 55^2} = 0.24 \]

Hence Pt = 0.12 % (Minimum)

\[ A_s = \frac{0.12}{100} \times 1000 \times 55 = 66\text{mm}^2 \]

Too Small Quantity
Provide 8 φ @ 150 c/c Spacing. (Since Spacing Shall not exceed 3d)

\[ A_{st} = 335 \text{mm}^2; \]
\[ P_t = \frac{335}{1000 \times 55} \times 100 = 0.61 \]
\[ \tau_c = 0.51 \text{MPa}(\text{Table} 61, \text{SP16}) \]
\[ \text{Shear Force} = 2.6 \times 0.75 = 1.95 \text{kN} \]
\[ \text{Shear Stress} = \frac{1.95 \times 1000}{1000 \times 55} \ll 0.51 \text{MPa} \]
\[ \frac{\text{Span}}{\text{Depth}} = \frac{75}{750} < 7 \]

Hence against deflection it is satisfactory.

**Design of Lintel:**

Ht of wall above Lintel = 1.8m > 0.866 x 1.5

The load is Triangular.

Clear Width = 1500mm

Bearing = 200mm

Effective span = 1700mm

\[ W = \frac{1}{2} \times 1.472 \times 1.7 \times 0.23 \times 20 = 5.76 \text{kN} \]
\[ M = \frac{WL}{6} = \frac{1.5 \times 1.7}{6} = 2.448 \text{kNm} \]
\[ d = \sqrt[300 \times 2.76]{2.45 \times 10^6} = 62 \text{mm} \]

Provide \( D = 115 \text{mm} \)

**FIGURE:**

B.M due to self wt. of sunshade = \( (2.6 \times 1.7)/12 = 0.63 \text{kNm} \)

Total moment = 3.08 kNm
\[ d = 115 - 25 - 5 = 85mm \]
\[ Mu = \frac{3.08 \times 10^6}{230 \times 85^2} = 1.85 \]
\[ Pt = 0.584 \]
\[ A_{st} = \frac{0.584 \times 230 \times 95}{100} = 114mm^2 \]

Provide 2 Nos. of 10 φ (100mm²)

\[ Pt = \frac{100 \times 100}{230 \times 85} = 0.51 \]
\[ ShearForce = \frac{3.36 \times 1.5}{2} = 2.52kN \]
\[ \tau_c = 0.46MPa \]
\[ \tau_{cal} = \frac{2.52 \times 1000}{230 \times 85} = 0.128 < 0.48MPa \]

Hence Provide Nominal Stirrup 6MS Bars @ 100mm c/c Spacing.

Cross section of Lintel

(ii)

**Example 8.10:**

*Design of a cantilever beam of span 3m with dead load including self weight 12 kN/m and imposed load 10kN/m. Concrete M20, Fe415.*

**Design Load:**

<table>
<thead>
<tr>
<th>Type</th>
<th>Load kN/m</th>
<th>Partial safety factor</th>
<th>Factored load kN/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>DL</td>
<td>12</td>
<td>1.5</td>
<td>18</td>
</tr>
<tr>
<td>IL</td>
<td>10</td>
<td>1.5</td>
<td>15</td>
</tr>
<tr>
<td>Total Pu</td>
<td></td>
<td></td>
<td>33</td>
</tr>
</tbody>
</table>

Design B.M. = \[ \frac{33 \times 3^2}{2} = 148.5kNm \]

\[ M_{u \text{lim}} = 2.76bd^2, \text{ Let } b=0.5d \]
148.5 \times 10^6 = 2.76 \times 0.75 \times d^3
\[ d = \sqrt[3]{\frac{148.5 \times 10^6}{2.76 \times 0.5}} = 475 \text{mm} \]

\[ b = 0.5 \times 475 = 237.5 \text{mm, say 230 mm} \]

148.5 \times 10^6 = 2.76 \times 230 \times d^2
\[ d = \sqrt{\frac{148.5 \times 10^6}{2.76 \times 230}} = 484 \text{mm} \]

\[ D = 484 + 25 + 10 = 529 \text{mm} \]

Let D = 530 mm and d = 495 mm

**Area of steel:**

\[
M_u = 0.87 f_y A_{st} \left[ d - \frac{f_y A_{st}}{f_y A_{st} b} \right]
\]

\[
148.5 \times 10^6 = 0.87 \times 415 \times A_{st} \left[ 495 - \frac{415 A_{st}}{20 \times 230} \right]
\]

\[
411300 = 495 A_{st} - 0.0902 A_{st}^2
\]

\[
A_{st}^2 - 54.88 A_{st} + 4559867 = 0
\]

\[
A_{st} = \frac{54.88 \pm \sqrt{54.88^2 - 4 \times 4559867}}{2} = 1021 \text{mm}
\]

**Curtailment:**

\[
P_{\text{max}} = \frac{0.955 \times 495 \times 230}{100} = 1087 \text{mm}^2
\]

Provide 2#20φ + 2#16φ = 1030 mm²

Let us curtail 2#16φ midspan and take 2#20φ throughout.

Capacity of section with 2#20φ rods @ midspan

\[
D @ \text{Midspan} = (530 + 230)/2 = 380
\]

\[
d = 380 - 35 = 345 \text{ mm}
\]

\[
M_u = 0.87 \times 415 \times 628 \left[ 345 - \frac{415 \times 628}{20 \times 230} \right] = 65.37 \text{kNm}
\]
Calculated $M_u$ @ Midspan = $\frac{33 \times 1.5^2}{2} = 37.125 kNm < 65.37 kNm$

Shear:

Design shear force @ support = $33 \times 3 = 99$ kN

Design shear strength of concrete = 0.6 MPa (Table 61 of SP: 16)

Shear capacity of concrete = $0.6 \times 230 \times 495 = 68.31$ kN

$V_{us} = 99 - 68.31 = 30.69$ kN.

Providing 2 legged 8 diameter stirrups, $S_v \approx 0.75d = 371$mm $\geq 300$.

Hence provide 2 legged 8 diameter stirrups @ 300 c/c

Provided $A_{sv} = 0.2217 \times 370 = 82 \text{mm}^2 < 100 \text{mm}^2$
SECTION

2-16φ (from taken support to nidspon then curtailed)

2-20φ

8 φ @ 370 c/c

2-12φ hanger rods
DISTANCE A
SKETCH SHOWING MOMENT TO WHICH BEAM @ SUPPORTS AND SPAN IS DESIGNED

HOGGING MOMENT @ SUPPORT A TO WHICH BEAM IS TO BE DESIGNED

HOGGING MOMENT @ SUPPORT B TO WHICH BEAM IS TO BE DESIGNED

HOGGING MOMENT AT SUPPORT B AS IN STAAD RESULT

MAXIMUM SAGGING SPAN MOMENT TO WHICH BEAM IS TO BE DESIGNED

DISTANCE A

EXAMPLE FOR NOTES

C/C OF COL ; 5960
5960
\[ \frac{5960}{12} = 497 \]
TOTAL DEPTH = 450

3104

B103 = STANDS FOR BEAM @ FIRST LEVEL AND 03 IS THE SERIAL NO

SUPPORT A

SUPPORT B

SKETCH SHOWING MOMENT TO WHICH BEAM @ SUPPORTS AND SPAN IS DESIGNED

NOTES;

(1) SECTION ARE RECTANGULAR

(2) TOTAL DEPTH = (C/C OF SUPPORTS / 12) AND RUUNED OFF TO NEAREST LOWER 50
**PRINCIPLES OF ADOPTED IN DESIGN AND DETAILING OF REINFORCEMENT**

**FOR PRIMARY AND CONTINUOUS BEAM**

- HANGER RODS 2-12Ø LAPPED TO MAIN REINFORCEMENT FOR 360mm
- REINFORCEMENT REQUIRED FOR BOND (OR) SAGGING MOMENT AT FACE A OR B WHICH EVER IS MAXIMUM
- REINFORCEMENT TO TAKE UP MAXIMUM SAGGING MOMENT IN SPAN
- REINFORCEMENT TO TAKE UP MAXIMUM HOGGING MOMENT AT SUPPORT (HIGHER OUT OF THAT REQUIRED FOR FACE A OR B)
- STIRRUP AT SUPPORTS
- STIRRUP AT SPAN
- NOT APPLICABLE FOR CONTINUOUS BEAMS

$L_d = 30 \text{TIMES } \phi \text{ OF BAR}$
CHAPTER 9  COMPRESSION MEMBERS

9.1 Definitions

9.1.1 Column or strut is a compression member, the effective length of which exceeds three times the least lateral dimension.

9.1.2 Short and slender Compression Member

A compression member may be considered as short when both the slenderness ratios $l_{ex}/D$ and $l_{ey}/b$ are less than 12:

Where,

- $l_{ex}$ - Effective length in respect of the major axis,
- $D$ - Depth in respect of the major axis
- $l_{ey}$ - Effective length in respect of the minor axis, and
- $b$ - Width of the member.

It shall otherwise be considered as a slender compression member.

9.1.3 Unsupported Length

The unsupported length, $l$, of a compression member shall be taken as the clear distance between end restraints except that:

a) In flat slab construction, it shall be clear distance between the floor and the lower extremity of the capital, the drop panel or slab whichever is the least.

b) In beam and slab construction, it shall be the clear distance between the floor and the underside of the shallower beam framing into the columns in each direction at the next higher floor level.

c) In columns restrained laterally by struts, it shall be the clear distance between consecutive struts in each vertical plane, provided that to be an adequate support, two such struts shall meet the columns at approximately the same level and the angle between vertical planes through the struts shall not vary more than 30° from a right angle. Such struts shall be of adequate dimensions and shall have sufficient anchorage to restrain the member against lateral deflection.

d) In columns restrained laterally by struts or beams with brackets used at the junction, it shall be the clear distance between the floor and the lower edge of the bracket, provided that the bracket width equals that of the beam struts and is at least half that of the column.

9.2 Effective length of Compression Members

In the absence of more exact analysis, the effective length $l_{ef}$ of columns may be obtained as described in Annex E of IS: 456/2000

9.3 Slenderness Limits for Columns

9.3.1 The unsupported length between end restraints shall not exceed 60 times the least lateral dimension of a column.

9.3.2 If, in any given plane, one end of a column is unrestrained, its unsupported length, $l$, shall not exceed $100 b^2/D$, where $b =$ width of that cross-section, and $D =$ depth of the cross-section measured in the plane under consideration.

9.4 Minimum Eccentricity

All columns shall be designed for minimum eccentricity, equal to the unsupported length of column / 500 plus lateral dimensions / 30, subject to a minimum of 20 mm. Where bi – axial bending is considered, it is sufficient to ensure that eccentricity exceeds the minimum about one axis at a time.

9.5 Design steps and salient points

Column or strut is a compression member, the effective length of which exceeds three times the least lateral dimension. (Clause 25.1.2)

9.5.1 Short and Slender Compression Members (Clause 25.1.2)

A compression member may be considered as short when both the slenderness ratios $L_{ex}/D$ and $L_{ey}/b$ are less than 12.
Where,
\[ L_{ex} = \text{effective length in respect of the major axis}, \]
\[ D = \text{depth in respect of the major axis}, \]
\[ L_{ey} = \text{effective length in respect of the minor axis}, \]
\[ b = \text{width of the member}. \]

It shall otherwise be considered as a slender compression member.

The member shall be designed by considering the assumptions given in Clause 39.1 of IS456:2000 and the minimum eccentricity. When the minimum eccentricity as per Clause 25.4 does not exceed 0.05 times the lateral dimension, the members may be designed by the following equation:

\[ P_U = 0.4 f_{ck} N_{Ac} + 0.67 f_y A_{Sc} \]

\[ P_U = \text{axial load on the member}, \]
\[ f_{ck} = \text{characteristic compressive strength of the concrete}, \]
\[ A_{c} = \text{Area of concrete}, \]
\[ f_y = \text{characteristic strength of the compression reinforcement}, \]
\[ A_{Sc} = \text{area of longitudinal reinforcement for columns}. \]

9.5.2 Types of problem

9.5.2.1 column section is known – to determine the safe load on the column

Step 1:

If the lateral reinforcements are only ordinary stirrups, calculate the load from equation (1) above. The strength of compression members with helical reinforcement satisfying the requirement of Clause 39.4.1 shall be taken as 1.05 times the strength of similar member with lateral ties. Clause 39.4.1 states that the ratio of the volume of helical reinforcement to the volume of the core shall not be less than 0.36\((A_g/A_c) - 1\)

\[ f_{ck}/f_y \]

\[ A_g = \text{gross area of the section}, \]
\[ A_c = \text{area of the core of the helically reinforced column measured to the outside diameter of the helix}, \]
\[ f_{ck} = \text{characteristic compressive strength of the concrete}, \]
\[ f_y = \text{characteristic strength of the helical reinforcement but not exceeding 415 N/mm}. \]

Step 2:

Calculate the effective length of the column and the ratio of effective length / lateral dimension. If this ratio is more than 12, the column is a slender one. The design of slender compression members shall be based on the forces and the moments determined from an analysis of the structure, including the effect of deflections on moments and forces.

9.5.2.2 To design a column when the load is known

Step 1:

Decide the concrete mix, steel grade.

Step 2:

The cross-sectional area of longitudinal reinforcement shall be not less than 0.8 percent nor more than 6 percent of the gross cross sectional area of the column.

**Note:** The use of 6 percent reinforcement may involve practical difficulties in placing and compacting of concrete; hence lower percentage is recommended. Where bars from the columns below have to be lapped with those in the column under consideration, the percentage of steel shall usually not exceed 4 percent.

Based on the above assume % reinforcement. From equation (1) above, we can calculate \(A_c\). The column may be square, rectangular, circular (or) octagonal. Accordingly find calculate the size of column. In the case of rectangular column the general engineering practice is to keep the ratio of width to depth less than 3.

Step 3:
Keep the vertical reinforcement to be equal to the assumed % and the lateral reinforcements 0.4% of volume of concrete for ordinary laterals (or) 0.5% to 1% if spirals are used. Find the minimum eccentricity and design using charts and tables in SP: 16.

**Step 4:**

Choose the diameter of the ties (or) spirals and calculate the pitch.

### 9.6 General notes on providing reinforcement for Columns

**9.6.1 Strength of columns:**

A column is generally reinforced with four (minimum) or more longitudinal bars, as shown in fig.1 (ii). The load that a column can carry without being overstressed is known as the strength of columns and is given by the sum of the loads on concrete and reinforcing bars.

The reinforcing bars should be a minimum of four with ordinary lateral reinforcements and a minimum of six with helical lateral. In six or more sided columns, the bars should be more, being a minimum of one of on for each side. The bars should be placed near the outside faces with a cover of at least 25mm or the diameter of the bar. When columns are to be protected against fire, a minimum cover of 50mm concrete is required.

**9.6.2 Lateral reinforcements**

There are two types of lateral reinforcements used. One consists of separate links or stirrups, made of wire, 5mm to 12mm diameter, bent round the longitudinal reinforcement at equal distances apart. These are known as ordinary lateral ties. Their main function is to prevent buckling of the longitudinal rods under loads. Let the maximum length of unsupported longitudinal bar be L and its diameter D. \( L = 16D \).

The spacing of the lateral ties must not be more than this distance if they are to serve their purpose, which is to support the longitudinal rods at intervals, and thus prevent their failure due to buckling. The pitch of the lateral ties should be not greater than the least lateral dimension of the column or 12 times the diameter of the longitudinal bar. The maximum allowed pitch is 300 mm. The volume of the lateral ties must be not less than 0.4 percent of the gross volume of the column. For the purpose of this rule both inside and outside ties (see fig.2) are included in the volume of ties.
As spiral reinforcement server to confine the concrete also, its pitch must be small and volume greater than the ordinary lateral ties. The pitch of the spiral shall be not more than 75 mm or \( \frac{1}{2} \) of the diameter of the core, whichever is smaller, and not less than 25 mm or 3 times the diameter of the spiral, whichever is greater. The volume of the spiral shall be \( \frac{1}{2} \) to 1 per cent of the volume of the concrete column.

The other type of lateral reinforcement consists of 5mm to 25mm bars wound round the longitudinal bars in a continuous spiral, and is therefore known as hoop or spiral reinforcements. The spiral reinforcement not only helps to support the longitudinal bars against buckling, but also forms a lateral...
support to the concrete inside the core. To secure this, code specifies that the longitudinal reinforcement should consist of at least six bars and they should be placed equidistantly around the inner circumference of the helical reinforcement. The concrete being thus confined can stand up to greater loads than when it is not confined. It is not possible to calculate from theory the exact amount of extra strength of a spirally wound column, but an empirical formula deduced from experiments can be used.

Extra strength = 1890 x $A_b$ ........................................... (i)

Where $A_b$ = equivalent area of spiral reinforcement, i.e., the volume of spiral reinforcement per cm length of the column. It must be noted that, though the formula involves the volume of spiral reinforcement, the lateral reinforcement does not carry any load. It enables the concrete to resist better. In spirally wound columns the concrete in the core only is assumed to take load.

### 9.6.3 Columns continuous over floors

When the columns are continuous over two floors but have different sizes, the reinforcement are arranged as shown.

1) The reinforcing bars for the top column starts only from the surface of the floor, and the reinforcement from the bottom are bent so that the neck of the bend may be within the slab portion so as to prevent its displacement under load. The length of a splice should be 24 times the dia of bar and the inclination shall be 1 in 6.

2) Splicing bars in compression may be done as shown below by (i) overlap of 24 d (ii) dowel bars of 48 d (iii) bringing the bars into contact with sleeve.

The third arrangement is not desired as it cannot resist bending moments.

![Splicing bars](image)

3) Columns are always subjected to bending in addition to compression due to eccentricity of load and must therefore be adequately designed for combined bending and compression.

### 9.6.4 Difficult Problems in Combined Bending and Direct Compression:

**Chimneys and Silos.**

Important applications of combined bending and compression occur in water tower columns, Silos and chimneys. The columns will usually be circular, hexagonal, octagonal, square and rectangular. In the case of a square section, bending about its diagonal due to wind loads has to be examined. Difficulties in calculation arise due to the variation of the width of the compressive concrete. Similarly, the determination of the neutral axis of a solid or hollow circular column with longitudinal bars placed all round involves tedious calculations. A useful approximation, made to reduce the difficulty in calculation, is to consider the longitudinal reinforcement replaced by circular ring of thickness such that its cross-sectional area is the as that of the rods. Also, in the case of circular section, it is far easier to represent the distances of the neutral axis in terms of the angles at the centre. In spite of all of these devices, the calculations remain complicated and the best way of dealing with these problems is by graphical determination of the neutral axis or using the coefficients given in many popular design manuals.

### 9.8.1 Uni-axial moment capacity about x axis.

\[
R_{ux} = \frac{d}{b_2}
\]

If $R_{ux} < 0.05$ \hspace{1cm} $R_0 = 0.05$

If 0.05 < $R_{ux} < 0.10$ \hspace{1cm} $R_0 = 0.10$

If 0.10 < $R_{ux} < 0.15$ \hspace{1cm} $R_0 = 0.15$

If 0.15 < $R_{ux} < 0.20$ \hspace{1cm} $R_0 = 0.20$
If $R_{uz} > 0.2$, Change clear cover (or) dia of bar (or) $b_z$ and redesign.

$P_{uz} x 1000/f_{ck} = A_c$, required area of concrete.

If $A_c > (b_x x b_z)$, increase the section and redesign.

From charts 43 to 46 (as applicable) read $M_{ux}/f_{ck} b_x b_z^2$ for the value of $P_{uz}/f_{ck} b_x b_z$ and $P_{uz}/f_{ck}$.

### 9.8.2 Moment capacity about x-axis

$$M_{ux1} = \{M_u x 10^5/f_{ck} b_x b_z^2\} x f_{ck} x b_x x b_z^2\} N mm$$

Uni-axial capacity of section about $zz$ axis.

$$R_{uz} = d''/b_x$$

If $R_{uz} < 0.05$ $R_{uz} = 0.05$

If $0.05 < R_{uz} < 0.10$ $R_{uz} = 0.10$

If $0.10 < R_{uz} < 0.15$ $R_{uz} = 0.15$

If $0.15 < R_{uz} < 0.20$ $R_{uz} = 0.20$

If $R_{uz} > 0.2$, Change clear cover (or) f of bar (or) size $b_z$ of column and proceed.

Read chart 43 to 46 (applicable) and obtain $M_{uz}/f_{ck} b_x b_z$ and $P_{uz}/f_{ck}$.

Calculate $M_{ux}/M_{ux1}$ and $M_{uz}/M_{uz1}$

Find permissible value of $M_{ux}/M_{ux1}$ corresponding to the value of $M_{uz}/M_{uz1}$ and $P_{uz}/P_{uz}$ chart 64

If actual value of $M_{ux}/M_{ux1}$ is greater than value read from chart, increase % reinforcement and redesign.

Dia of main reinforcement proposed $f_c$ in mm

Area of bar $= p x f_c^2/4$ sq mm $= A_{rt}$

No of bars $N_e = (A_s/A_{rt})$ rounded off to next higher integer

Area of steel provided $A_{sp} = (N_e X_{art})$ sq mm

% of steel provided $= A_{sp} x 100/ b_x x b_z = P_p$

(Preferable that $P_p < 3\%$)

### Checking the section

$\left( P_{uz}/f_{ck} \right)$ =

Referring to chart 43 to 46 (applicable)

Find $M_{ux}/f_{ck} b_x b_z^2$

$M_{ux1} = (M_u/f_{ck} b_x b_z^2) x f_{ck} x b_x x b_z^2$

$M_{uy1} = (M_u/f_{ck} b_x b_z^2) x f_{ck} x b_x^2 x b_z$

$P_{uz} = 0.45 f_{ck} (b_x b_z - A_{sp}) + 0.75 f_c A_{sp}$

Calculate

$P_{uz}/P_{uz}$

$M_{ux}/M_{ux1}$

$M_{uy}/M_{uy1}$

Find from chart 64, the permissible value of

$M_{ux}/M_{ux1}$ corresponding to the value of $M_{uz}/M_{uz1}$ and $P_{uz}/P_{uz}$

If this is less than the actual $M_{ux1}/M_{ux1}$ satisfactory.

Lateral reinforcement $(l_f = f_c/4)$

Minimum diameter $(f_c/4) mm$

But greater than (or) equal to 6 mm rounded off to higher value 6 mm, 8 mm, 10 mm, 12 mm

Maximum spacing of lateral ties –least of

i) 16 f

ii) 48 f

iii) And least lateral dimension.

### 18.3 Design of Column (Limit state method)
Column designation:-
Design section @ EL + m
Depth of the beam having lesser depth out of all the beams connected to top of column is considered for arriving at the height of column.
Elevation of top of column up to bottom of beam having depth equal to ‘d_{bs}’ = E_{1t} in M
Elevation of bottom of column: = E_{1b} in M
Factored axial load = P_{u} kN
Factored moment = M_{ux} kNm
Factored moment = M_{uz} kNm
Size of column along x = b_{x} mm
Size of column along Z = b_{z} mm
Characteristic strength of concrete = f_{c}k MPa
Characteristic strength of steel = f_{y} MPa
Unsupported length of column = l_{u} = (E_{1t} x 1000 – E_{1b} x 1000 – d_{bs}) mm
(IS :456 cl 24. 1.3)
e_{x min} = ((l_{u} / 500 ) + (b_{x} / 30)) mm
If e_{x min} < 20 mm, then e_{min} = 20 mm.
M_{xe} = (P_{u} x e_{x min}/1000 ) kNm
Ez min = ((l_{u} / 500) +(b_{z} / 30)) mm
If e_{z min} < 20 mm, then e_{z min} = 20 mm.
M_{ze} = (P_{u} x e_{z min}/1000) kNm.
If M_{ux} < M_{xe} ; M_{x} = M_{xe}
If M_{uz} < M_{ze} ; M_{z} = M_{ze}
Considering distribution of steel on all four sides, P_{1.2} (assume)
P/f_{c}k = 1.2/f_{c}k
Uni-axial bending capacity of the section about ‘x’ axis.
Clear cover = 25mm
Diameter of bar proposed = f_{c} mm
d’ = (25 + f_{c}/2) mm.
COLUMN DESIGNATION AND ORIENTATION

COLUMN DESIGNATION K4

DIRECTION OF LONG FRAME

VIEW-B

XX - MAJOR AXIS OF COLUMN
YY - MINOR AXIS OF COLUMN

VIEW-A
CHAPTER 10 REQUIREMENTS GOVERNING REINFORCEMENT AND DETAILING
10.1 General

Simultaneous use of two different types of grades of steel for main and secondary reinforcement respectively is permissible. Bars larger than 32 mm diameter shall not be bundled except in columns. Detailing for Earthquake resistant construction IS: 13920 & IS: 4326.

10.2 Development length of bars

\[ L_d = \frac{\varphi \sigma_s}{4 \eta_{bd}} \]

Where
- \( \varphi \) - Nominal diameter of the bar (mm)
- \( \sigma_s \) - Stress in bar at the section considered at design load (N / mm²)
- \( \eta_{bd} \) - design bond stress (N / mm²)

Notes: The development length (\( L_d \) in mm) includes anchorages values of hooks in tension reinforcement.

The bond strength of concrete on unit surface area of steel is called bond stress. Development length is the minimum length of bar required to embedded into the concrete for developing the required axial stress in the bar.

Axial stress in bar = \( \sigma_s = \frac{T}{(\pi \varphi^2 / 4)} = \frac{4T}{(\pi \varphi^2)} \)

\( T = (\pi \varphi^2 / 4) \times \sigma_s \)

Bond stress (permissible) = \( \eta_{bd} \)

Bond stress of concrete = \( \pi \varphi \times L_d \times \eta_{bd} = (\pi \varphi^2 / 4) \times \sigma_s \)

\[ L_d = (\pi \varphi^2 / 4) \times \sigma_s / \eta_{bd} \]

10.3 Bends and hooks

As per IS : 2502.

10.4 Curtailment of Tension reinforcement in flexural members (Clause 26.2.3 IS :456/2000)

10.4.1 For curtailment, reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of the member or 12 times the bar diameter, whichever is greater except at simple support or end of cantilever. In addition 10.4.2 to 10.4.5 shall also be satisfied.

Note: A point at which reinforcement is no longer required to resist flexure is where the resistance moment of the section, considering only the continuing bars, is equal to the design moment.

10.4.2 Flexural reinforcement shall not be terminated in a tension zone unless any one of the following conditions is satisfied:
   a) The shear at the cut – off point does not exceed two – thirds that permitted, including the shear strength of web reinforcement provided.
   b) Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from the cut – off point equal to three – fourths the effective depth of the member. The excess stirrup area shall be not less than 0.4 bs / fy, where b is the breadth of beam, s is the spacing and fy is the characteristic strength of reinforcement in N / mm². The resulting spacing
shall not exceed $d/8$ where $B_b$ is the ratio of the area of bars cut-off to the total area of bars at the sections, and $d$ is the effective depth.

c) For 36 mm and smaller bars, the continuing bars provide double the area required for flexure at the cut-off point and the shear does not exceed three-fourths that permitted.

### 10.4.3 Positive moment reinforcement

a) At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of the member in to the support, to a length equal to $L_d / 3$.

b) When a flexural member is part of the primary lateral load resisting system, the positive reinforcement required to be extended into the support as described in (a) shall be anchored to develop its design stress in tension at the face of the support.

c) At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that $L_0$ computed for $f_d$ by 10.2 does not exceed:

$$(M_1 / V) + L_0$$

Where,

$M_1$ – Moment of resistance of the section assuming all reinforcement at the section to be stressed to $f_d$;

$f_d$ – 0.87 $fy$ in the case of limit state design and the permissible stresses $\sigma_{st}$ in the case of working stress design;

$V$ – Shear force at the section due to design loads;

$L_0$ – Sum of the anchorage beyond the center of the support and the equivalent anchorage value of any hook or mechanical anchorage at simple support; and at a point of inflection, $L_0$ is limited to the effective depth of the members or $12 \varphi$ is greater; and $\varphi$ - diameter of bar.

The value of $(M_1 / V)$ in the above expression may be increased by 30 percent when the ends of the reinforcement are confined by a compressive reaction.

### 10.4.4 Negative Moment reinforcement

At least one-third of the total reinforcement provided for negative moment at the support shall extend beyond the point of inflection for a distance not less than the effective depth of the member of 12 $\varphi$ or one-sixteenth of the clear span whichever is greater.

### 10.4.5 Curtailment of bundled bars

Bars in a bundle shall terminate at different points spaced apart by not less than 40 times the bar diameter except for bundles stopping at a support.

### 10.5 Special Members

Adequate end anchorage shall be provided for tension reinforcement in flexural members where reinforcement stress is not directly proportional to moment, such as sloped, stepped, or tapered footings; brackets; deep beams; and members in which the tension reinforcement is not parallel to the compression face.

### 10.6 Reinforcement Splicing

Where splices are provided in the reinforcing bars, they shall as far as possible be away from the sections of maximum stress and be staggered. It is recommended that splices in flexural members should not be at sections where the bending moment is more than 50 percent of the moment of resistance; and not more than half the bars shall be spliced at a section.

Where more than one-half of the bars are spliced at a section or where splices are made at points of maximum stress, special precautions shall be taken, such as increasing the length of lap and/or using spirals or closely-spaced stirrups around the length of the splice.

#### 10.6.1 Lap splices

a) Lap splices shall not be used for bars larger than 36 mm; for larger diameters, bars may be welded. In cases where welding is not practicable, lapping of bars larger than 36 mm may be permitted, in which case additional spirals should be provided around the lapped bars.
b) Lap splices shall be considered as staggered if the center to center distance of the splices is not less than 1.3 times the lap length calculated as described in (c).

c) Lap length including anchorage value of hooks for bars in flexural tension shall be \( L_d \) or \( 30\varphi \) whichever is greater and for direct tension shall be \( 2L_d \) or \( 30 \varphi \) whichever is greater. The straight length of the lap shall not be less than 15 \( \varphi \) or 200 mm. The following provisions shall also apply:

Where lap occurs for a tension bar located at:

1) Top of a section as cast and the minimum cover is less than twice the diameter of the lapped bar, the lap length shall be increased by a factor of 1.4.
2) Corner of a section and the minimum cover to either face is less than twice the diameter of the lapped bar or where the clear distance between adjacent laps is less than 75 mm or 6 times the diameter of lapped bar, whichever is greater, the lap length should be increased by a factor of 1.4.

Where both condition (1) and (2) apply, the lap length should be increased by a factor of 2.0.

Note: Splices in tension members shall be enclosed in spirals made of bars not less than 6 mm diameter with pitch not more than 100 mm.

d) The lap length in compression shall be equal to the development length in compression, calculated as described, in 10.2 but not less than 24 \( \varphi \).

e) When bars of two different diameters are to be spliced, the lap length shall be calculated on the basis of diameter of the smaller bar.

f) When splicing of welded wire fabric is to be carried out, lap splices of wires shall be made so that overlap measured between the extreme cross wires shall be not less than the spacing of cross wires plus 100 mm.

g) In case of bundled bars, lapped splices of bundled bars shall be made by splicing one bar at a time; such individual splices within a bundle shall be staggered.

10.6.2 Strength of welds

The following values may be used where the strength of the weld has been proved by tests to be at least as great as that of the parent bar.

a) Splices in compression – For welded splices and mechanical connection, 100 percent of the design strength of joined bars.

b) Splices in tension

1) 80 percent of the design strength of welded bars (100 percent if welding is strictly supervised and if at any cross-section of the member not more than 20 percent of the tensile reinforcement is welded)
2) 100 percent of design strength of mechanical connection.

10.6.3 End – bearing splices

End bearing splices shall be used only for bars in compression. The ends of the bars shall be square cut and concentric bearing ensured by suitable devices.

10.7 Spacing of Reinforcement

10.7.1 For the purpose of this clause, the diameter of a round bar shall be its nominal diameter, and in the case of bars which are not round or in the case of deformed bars or crimped bars, the diameter shall be taken as the diameter of a circle giving an equivalent effective area. Where spacing limitations and minimum concrete cover are based on bar diameter, a group of bars bundled in contact shall be treated as a single bar of diameter derived from the total equivalent area.

10.7.2 Minimum Distance Between Individual Bars

The following shall apply for spacing of bars:

a) The horizontal distance between two parallel main reinforcing bars shall usually be not less than the greatest of the following:

1) The diameter of the bar if the diameters are equal,
2) The diameter of the larger bar if the diameters are unequal, and
3) 5 mm more than the nominal maximum size of coarse aggregate.

Note: This does not preclude the use of larger size of aggregates beyond the congested reinforcement in the same member; the size of aggregates may be reduced around congested reinforcement to comply with this provision.

b) Greater horizontal distance than the minimum specified in (a) should be provided wherever possible. However when needle vibrators are used the horizontal distance between bars of a
group may be reduced to two – thirds the nominal maximum size of the coarse aggregate, provided that sufficient space is left between groups of bars to enable the vibrator to be immersed.

Where there are two or more rows of bars, the bars shall be vertically in line and the minimum vertical distance between the bars shall be 15 mm, two – thirds the nominal maximum size of aggregate or the maximum size of bars, whichever is greater.

**10.7.3 Maximum Distance between Bars in Tension**

Unless the calculation of crack widths shows that a greater spacing is acceptable, the following rules shall be applied to flexural members in normal internal or external conditions of exposure.

a) **Beams** – The horizontal distance between parallel reinforcement bars, or groups, near the tension face of a beam shall not be greater than the value given in table 10.1 depending on the amount of redistribution carried out in analysis and the characteristic strength of the reinforcement.

b) **Slabs**

   1) The horizontal distance between parallel main reinforcement bars shall not be more than three times the effective depth of solid slab or 300 mm whichever is smaller.

   2) The horizontal distance between parallel reinforcement bars provided against shrinkage and temperature shall not be more than five times the effective depth of a solid slab or 450 mm whichever is smaller.

**Table 10.1 Clear Distance Between Bars**

<table>
<thead>
<tr>
<th>Fy (N/mm²)</th>
<th>Percentage Redistribution to or from Section Considered</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-30</td>
</tr>
<tr>
<td>Clear Distance Between Bars</td>
<td></td>
</tr>
<tr>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
<td>250</td>
<td>215</td>
</tr>
<tr>
<td>415</td>
<td>125</td>
</tr>
<tr>
<td>500</td>
<td>105</td>
</tr>
</tbody>
</table>

**Note:** The spacing given in the table are not applicable to members subjected to particularly aggressive environment unless in the calculation of the moment of resistance, fy has been limited to 300 N/mm² in limit state design and ost limited to 165 N/mm² in working stress design.

**10.8 Nominal Cover to Reinforcement**

**10.8.1 Nominal Cover**

Nominal cover is the design depth of concrete cover to all steel reinforcement, including links. It is the dimension used in design and indicated in the drawings. It shall be not less than the diameter of the bar.

**10.8.2 Nominal Cover to Meet Durability Requirement**

Minimum values for the nominal cover of normal weight aggregate concrete which should be provided to all reinforcement, including links depending on the condition of exposure described in 8.2.3 shall be as given in Table 10.2.

**Note:**

1) For main reinforcement up to 12mm diameter bar for mild exposure the nominal cover may be reduced by 5mm.

2) Unless specified otherwise, actual concrete cover should not deviate from the required nominal cover by + 10 mm/0 mm.

3) For exposure condition ‘severe’ and ‘very severe’, reduction of 5 mm may be made, where concrete grade is M35 and above.

**Table 10.2 Nominal Cover to Meet Durability Requirements**

<table>
<thead>
<tr>
<th>Exposure</th>
<th>Nominal concrete Cover in mm not less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild</td>
<td>20</td>
</tr>
<tr>
<td>Moderate</td>
<td>30</td>
</tr>
<tr>
<td>Severe</td>
<td>45</td>
</tr>
<tr>
<td>Very severe</td>
<td>50</td>
</tr>
<tr>
<td>Extreme</td>
<td>75</td>
</tr>
</tbody>
</table>
10.8.2.1 However for a longitudinal reinforcing bar in a column nominal cover shall in any case not be less
than 40 mm, or less than the diameter of such bar. In the case of columns of minimum dimension of 200
mm or under, whose reinforcing bars do not exceed 12 mm, a nominal cover of 25 mm may be used.

10.8.3 For footings minimum cover shall be 50 mm.

10.9 Nominal Cover to Meet specified period of Fire Resistance

Minimum values of nominal cover of normal – weight aggregate concrete to be provided to all
reinforcement including links to meet specified period of fire resistance shall be given in Table 10.3

Table 10.3 Nominal Cover to Meet specified period of Fire Resistance

<table>
<thead>
<tr>
<th>Fire resistance</th>
<th>Nominal Cover</th>
<th>Slabs</th>
<th>Ribs</th>
<th>Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simply supported</td>
<td>Continuously supported</td>
<td>Simply supported</td>
<td>Continuously supported</td>
</tr>
<tr>
<td>$h$</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
<td>0.5</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>1.0</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>1.5</td>
<td>20</td>
<td>20</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>2.0</td>
<td>40</td>
<td>30</td>
<td>35</td>
<td>25</td>
</tr>
<tr>
<td>3.0</td>
<td>60</td>
<td>40</td>
<td>45</td>
<td>35</td>
</tr>
<tr>
<td>4.0</td>
<td>70</td>
<td>50</td>
<td>55</td>
<td>45</td>
</tr>
</tbody>
</table>

Note:
1) The nominal covers given relate specifically to the minimum member dimensions given in
Fig.1/IS456:2000
2) Cases that lie below the bold line require attention to the additional measures necessary to reduce
the risks of spalling

10.10 Requirements of reinforcement for structural members

10.10.1 Beams

10.10.1.1 Tension reinforcement

(a) Minimum reinforcement

$$\frac{A_s}{bd} = 0.85 / \frac{f_y}{b}\,$$

As = minimum area of reinforcement in sqmm
b = breadth of beam in mm
d = effective depth in mm
f_y = characteristic strength of reinforcement in N / mm².

(b) maximum reinforcement 0.04bd

10.10.1.2 Compression reinforcement

The maximum area of compression reinforcement shall not exceed 0.04bd

10.10.1.3 Side face reinforcement

Where the depth of the web in a beam exceeds 750 mm, side face reinforcement shall be provided
along the two faces. The total area of such reinforcement shall be not less than 0.1% of the web area and
shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is
less.
10.10.1.4 Maximum spacing of shear reinforcement

The maximum spacing of shear reinforcement measured along the axis of the member shall not exceed 0.75d for vertical stirrups and d for inclined stirrups @ 45 degrees. In no case, the spacing shall exceed 300 mm.

10.10.1.5 Minimum shear reinforcement

\[(\frac{Asv}{bsv}) > or = (\frac{0.4}{0.87fy})\] in the form of stirrups

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asv</td>
<td>Total C.S area of stirrup legs effective in shear (mm²)</td>
</tr>
<tr>
<td>Sv</td>
<td>Stirrups spacing along the length of the member (mm)</td>
</tr>
<tr>
<td>b</td>
<td>breadth of the beam or breadth of the web of flanged beam (mm)</td>
</tr>
<tr>
<td>fy</td>
<td>Characteristic strength of the stirrup reinforcement in N / mm² which shall not be taken greater than 415 N / mm².</td>
</tr>
</tbody>
</table>

10.10.1.6 Minimum Reinforcement

For slabs 0.15% Mild steel
For slabs 0.12% high strength deformed bars.

10.10.1.7 Maximum diameter

The diameter of reinforcing bars shall not exceed one eighth of the total thickness of slab.

10.10.2 Columns

10.10.2.1 Longitudinal reinforcement

Not less than 0.8% and not more than 6% of the gross C.S area of the column. (Normally not to exceed 4%)
If the column provided has larger area than that required, consider only the required area of column.
Minimum no. of bars: four
Minimum diameter of bars: 12 mm
Spacing of longitudinal bars measured along the periphery of the column shall not exceed 300 mm.
For pedestals; provide minimum 0.15% of C.S area of pedestal.
Pedestal is a compression member, the effective length of which does not exceed three times the least lateral dimension.

10.10.2.2 Transverse reinforcement

a) General – A reinforcement concrete compression member shall have transverse or helical reinforcement so disposed that every longitudinal bar nearest to the compression face has effective lateral support against buckling subject to provisions in (b). The effective lateral support is given by transverse reinforcement either in the form of circular rings capable of taking up circumferential tension or by polygonal links (lateral ties) with internal angles not exceeding 135 degrees. The ends of the transverse reinforcement shall be properly anchored

b) Arrangement of transverse reinforcement

1) If the longitudinal bars are not spaced more than 75 mm on either side, transverse reinforcement need only to go round corner and alternate bars for the purpose of providing effective lateral supports
2) If the longitudinal bars spaced at a distance of not exceeding 48 times the diameter of the tie are effectively tied in two directions, additional longitudinal bars in between these bars need to be tied in one direction by open ties.
3) Where the longitudinal reinforcing bars in a compression member are placed in more than one row, effective lateral support to the longitudinal bars in the inner rows may be assumed to have been provided if:
   (i) Transverse reinforcement is provided for the outer-most row in accordance with 10.10.2.2, and
   (ii) No bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row.
4) Where the longitudinal bars in a compression member are grouped (not in contact) and each group adequately tied with transverse reinforcement in accordance with 10.10.2.2, the transverse reinforcement for the compression member as a whole may be provided on
the assumption that each group is a single longitudinal bar for purpose of determining the pitch and diameter of the transverse reinforcement in accordance with 10.10.2.2. The diameter of such transverse reinforcement need not, however, exceed 20 mm.

c) **Pitch and diameter of lateral ties**

1) **Pitch** – The pitch of transverse reinforcement shall be not more than the least of the following distances:

   (i) The least lateral dimension of the compression members.
   
   (ii) Sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and
   
   (iii) 300mm

2) **Diameter** – The diameter of the polygonal links or lateral ties shall be not less than one fourth of the diameter of the largest longitudinal bar, and in no case less than 16 mm.

---

**CHAPTER 11.0 EXPANTION JOINTS**

The structures adjacent to the joint should preferably be supported on separate columns or walls but not necessarily on separate foundation.

Normally structures exceeding 45m in length are designed with one or more expansion joints (Refer IS: 3414)
CHAPTER 12.0 STAIRS

12.1 Effective span of stairs

The effective span of stairs without stringer beams shall be taken as the following horizontal distances:

a) Where supported at top and bottom risers by beams spanning parallel with the risers, the distance centre to centre of beams (See Fig-12.3).

b) Where spanning on to the edge of a landing slab, which spans parallel, with the risers (see Fig 12.4), a distance equal to the going of the stairs plus at each end either half the width of the landing or one metre, whichever is smaller; and

c) Where the landing slab spans in the same direction as the stairs, they shall be considered as acting together to form a single slab and the span determined as the distance center to center of the supporting beams or walls, the going being measured horizontally. (See Fig – 12.5).

12.2 Distribution of Loading on Stairs

In the case of stairs with open wells, where spans partly crossing at right angles occur, the load on areas common to any two such spans may be taken as one half in each direction as shown in Fig 18.

where flights or landings are embedded into walls for a length of not less than 110mm and are designed to span in the direction of the flight, a 150 mm strip may be deduced from the loaded area and the effective breadth of the section increased by 75mm for purposes of design (see Fig 19/IS456:2000).

12.3 Depth of Section

The depth of section shall be taken as the minimum thickness perpendicular to the soffit of the staircase.

12.4 Design of Stair Cases
Staircase is a sloping beam with step tread arrangement spanning from one floor to another floor. Slope of staircase 30° to 42°, Tread 230 to 300mm, Risers 150 to 190mm.

<table>
<thead>
<tr>
<th>Tread(mm)</th>
<th>230</th>
<th>250</th>
<th>280</th>
<th>300</th>
<th>330</th>
<th>350</th>
<th>380</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risers(mm)</td>
<td>190</td>
<td>170</td>
<td>160</td>
<td>150</td>
<td>140</td>
<td>120</td>
<td>110</td>
</tr>
</tbody>
</table>

Horizontal loads on hand rails parapets for stairs may be taken at 75 kg/m run.

**Most common types are:**

(I) A sloping slab spanning from one floor to another floor (or) from end to a landing and supported on the two sides walls or on sloping stringer beam.

(II) Separate slabs for each step attached to one central sloping beam.

(III) Steps cantilever from a side wall (one end fixed into the wall) where the wall is of sufficient thickness.

(IV) Spiral stairs with slabs cantilevered out from a central column.

(V) Free spanning spiral stairs.

**Imposed load** allowed on is 300kg/sqm for residential / office buildings and 500kg/sqm for public buildings like schools, assembly halls and warehouse. Service stairs for maintenance can be designed for an imposed load of 150 kg/sqm. Cantilever steps are designed for 150 kg concentrated load @ the free end of each step. Each step shall be embedded for 250 to 300 mm inside wall for anchorage.

Bending moment on waist slab WL/10 where ends are built into walls and WL/12 where ends are monolithic with the transverse beams @ bottom and top.

Slope of staircase 30° to 42°, Tread 230 to 300mm, Risers 150 to 190 mm.

<table>
<thead>
<tr>
<th>Tread (mm)</th>
<th>230</th>
<th>250</th>
<th>280</th>
<th>300</th>
<th>330</th>
<th>350</th>
<th>380</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risers (mm)</td>
<td>190</td>
<td>170</td>
<td>160</td>
<td>150</td>
<td>140</td>
<td>120</td>
<td>110</td>
</tr>
</tbody>
</table>

Horizontal loads on hand rails or parapets for stairs may be taken at 75 kg/m run.

**12.5 Types of Staircases:**

**Generally adopted dimensions are as below:**

Rise 150 to 200 mm

Tread 225 to 300 mm

Width : 900 to 1200 for residential and 1200 to 1800 public buildings

Number of steps per flight shall not exceed 12
Effective span:

When supported at top and bottom risers by beams as shown in figure 1.0

Figure 1.0

Effective span = Horizontal distance centre to centre of beams

(a) when supported at top and bottom by landing slabs which span perpendicular to the flight as shown in figure 2.0

Effective Span = Going of stair + Half the width of landings.

Figure 2.0

(c) When the landing slab also spans in the direction of flight.

The flight slab as well as the landing slab is acting as a single slab.

Effective Span = the horizontal distance c/ c of supporting walls or beams as shown.
(c) **Stairs spanning perpendicular to the flight.**

The waist slab is supported by inclined edge beams (or ) walls along both of its edges as shown in figure 4.0.

Effective Span = Distance between c/c of supports

---

**Example 12.1: Design of flight slab**
Design data

Each flight has 12 steps.
Tread 259mm.
Rise 150mm
Imposed load = 5 kN/m²
Density of brick work = 20 kN /cum.
Weight of floor finish shall be considered as 1 kN/m² of plan area.
Concrete M20 f_y = 415.
Flight slab is not support by side walls.

\[
\text{Effective span : } 2500 + \frac{1000}{2} + \frac{1000}{2} = 3500 \text{ mm}
\]

Consider 1 m width of slab.
Imposed load = 5 kN / m²
Weight of steps = \( \frac{1000}{250} \times 0.25 \times 0.15 \times 20 = 1.5 \text{ kN/m} \)

Thickness of waist slab = \( \frac{3500}{20} = 175 \text{ mm} \)

Self weight of slab / m² of inclined area = \( 1 \times 0.175 \times 25 = 4.125 \text{ kN/m²} \)
Self weight of slab / m of horizontal length = \( \frac{0.25}{2} + \frac{0.15}{2} = 4.125 \times \frac{0.25}{0.25} = 4.81 \text{ kN/m} \)

Weight of finish = 1.00 kN /m
Total load = 5 + 1.5 + 4.81 + 1.1 = 12.31 kN /m
The same loading is assumed in the landing also.
Partial safety factor = 1.5
Design load = 1.5 x 12.31 = 18.465 kN/m

\[
18.465 \times 3.5^2
BM = Mu = \frac{28.2745 \text{ kNm}}{8}

M_y limit = 2.76 bd^2
28.2745 \times 16.6 = 2.76 \times 1000 \times d^2
\]

\[
d = \sqrt{\frac{28.2745 \times 10^6}{2.76 \times 1000}} = 101.20 \text{ mm}
\]

D = 175 mm  \( d = 175 - 15 - 6 = 154 \text{ mm} \)
Main reinforcement

\[
MR = 0.87 f_y A_{st} \left( \frac{d - \frac{d}{2}}{f_y A_{st}} \right) = 0.87 \times 415 \times A_{st} \left[ \frac{154 - \frac{d}{2}}{20 \times 1000} \right] = 28.2745 \times 10^6 \text{Nmm}
\]

\[
154 A_{st} - 0.02075 A_{st}^2 = 78311.87
\]

\[
A_{st} = 7421.69 A_{st} + 3774066 = 0
\]

\[
7421.69 \sqrt{(7421.69 \times 2 - 4 \times 3774066)}
\]

\[
A_{st} = \frac{549 \text{mm}^2}{2}
\]

Number of 12 mm dia bars required / 1.2 m width = \( \frac{549 \times 1.2}{113} \approx 5.8 \)

Provide 6 Numbers of 12 mm dia bars giving 678 mm\(^2\) per 1200 mm width.

Provided reinforcement per 1000 mm width = \( \frac{(678/1200) \times 1000}{565} = 565 \text{mm}^2 \).

Minimum area of steel

\[
A_{st} = \frac{0.85}{bd} = \frac{0.85}{f_y}
\]

\[
A_{st} = \frac{0.85}{1000 \times 154} = 315 \text{mm}^2 < 565 \text{mm}^2 \text{ provided}
\]

Hence satisfactory.

Distribution

\[
\frac{0.12}{100} \times A_{st} \text{ required} = \frac{0.12}{100} \times 1000 \times 175 = 210 \text{mm}^2
\]

Provide 8 mm dia bars @ 240 mm c/c

Check for shear

\[
18.465 \times 3.5
\]

Shear force = \( \frac{31.314 \text{kN}}{2} \)

\[
\tau_v = \frac{32314}{1000 \times 154} = 0.21
\]

To find shear strength

\[
A_{st} \text{ per 1000 mm width of slab} = \frac{565 \text{mm}^2}{655 \times 100}
\]

\[
\% A_{st} = \frac{0.37}{1000 \times 154} \times 0.12 = 0.42 > 0.21
\]

Hence satisfactory.

Check for stinffness

Basic span depth ratio = 20.

\% reinforcement 0.37%
Modification factor

\[ f_s = 0.58 \times 415 \times \left(\frac{549}{565}\right) = 234 \]

Modification factor 1.4 (From figure 4 of IS: 456:2000)

Effective depth required for stiffness

\[ \frac{3500}{20 \times 1.4} = 125\text{mm} < \text{Less than provided. Hence safe against deflection.} \]

**CHAPTER 13.0 FOOTINGS**

13.1 **General**

Safe bearing capacity of soil - refer IS:1904

Thickness at the edge of footing shall not be less than 150mm for footings on soils, not less than 300 mm above the tops of piles for footing on piles.
\[ \tan \alpha \text{ not less than } \frac{0.9}{[(100 \times qa + 1)/ fck]} \]

- \( q_a \): Calculated maximum bearing pressure at the base of the pedestal in N/mm² and
- \( fck \): Characteristic strength of concrete @ 28 days in N/mm².

### 1.1 Isolated footing

*A column is subjected to dead and imposed load of 600kN. The size of the column is 230 x 450mm. Safe bearing capacity of soil is 200kN/sq.m. Concrete M 20 and Steel Fe 415.*

**Load from column**

600kN

**Add 10% for difference in weight of footing and soil replaced**

60kN

**Total load on soil**

660kN

**Area of footing required**

\[ \frac{660}{200} = 3.3 \text{sq.m.} \]

Assuming the footing has the same ratio of length to width as those for column.

\[ \frac{450}{230} = 2 \]

Footing length = 2 x footing width

\[ L = 2B \]

Area = \( L \times B = 2B \times B = 2B^2 \)

\[ \therefore 2B^2 = 3.3 \text{m.} \]

\[ B = \sqrt{3.3/\sqrt{2}} = 1.28 \text{m} \]

Let the footing size be 1.25m x 2.5m

\[ q = \frac{600}{1.25 \times 2.5} = 192 \text{kN/sq.m} < 200 \text{kN/sq.m.} \]

**Design of the section**

Factored \( q = 1.5 \times 192 = 288 \text{kN/sq.m.} \)

Maximum factored BM at XX = \((288 \times 1.025^2 \times 1.25 / 2)\)

\[ = 189.1125 \text{kNm} \]

Moment capacity of Trapezoidal section

\[ Mr = kNb + kN_f (b - b_i) d^2 \sigma_{acb} \]

**Design parameter for balanced section**
<table>
<thead>
<tr>
<th>Fy</th>
<th>$x_c/d$</th>
<th>kN</th>
<th>kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.531</td>
<td>0.149</td>
<td>0.03</td>
</tr>
<tr>
<td>415</td>
<td>0.479</td>
<td>0.138</td>
<td>0.025</td>
</tr>
<tr>
<td>500</td>
<td>0.456</td>
<td>0.133</td>
<td>0.023</td>
</tr>
</tbody>
</table>

In our case

$kN = 0.138; \quad kN_1 = 0.025; \quad b_1 = 230\text{mm};$

$b = 1250\text{mm}; \quad \sigma_{cb} = 20 \text{N/mm}^2$

$M_r = 189.11\text{kNm}$

$M_r = 0.138 \times 230 + 0.025 (1250 - 230) d^2 \times 20$

$= 189.11 \times 10^6$

$= 31.74 + 510 d^2$

∴ $d = \sqrt{[(189.11 \times 10^6 - 31.74)/510]} = 525\text{mm}$

$D = 522 + 10 + 40 = 572\text{mm}$

Keep $D = 575\text{mm} \quad d = 525\text{mm}$

**Check for shear capacity**

The critical section where shear failure is likely to occur is at a distance $d/2$ the face of the column.

The periphery of the shear boundary is $b_0 = 2 (755 + 975) = 3460\text{mm}$

Net shear force acting on the periphery of critical shear boundary is

$$288 \times \frac{(1250 - 755)}{1000} \times \frac{(2500 - 975)}{1000} = 288 \times 0.495 \times 1.525 = 217\text{kN}$$

The effective depth at $d/2$ distance is
\[ d_s = 150 + \frac{425}{1025} \times \frac{525}{2} = 466\text{mm} \]

Nominal shear stress at the critical zone is
\[ \tau_v = \frac{217 \times 1000}{3460 \times 466} = 0.1345\text{N/mm}^2 \]

The shear strength of M20 concrete for slab is \( \tau_c = 0.25 \sqrt{20} = 1.372\text{MPa} > 0.1345\text{MPa} \)

The section is safe against shear failure including transverse shear.

**Design of reinforcement**

Area of reinforcement for a balanced trapezoidal section is

\[ A_{st} = \frac{1.15M}{jfd} = \frac{1.15 \times 189.11 \times 10^6}{0.798 \times 525 \times 415} = 1250.8\text{mm}^2 \]

Provide 12 numbers of 12Ø giving \( A_{st} = 1357\text{mm}^2 \).

**Design of reinforcement in the short span direction**

\[ d_1 = d - 12 = 525 - 12 = 513\text{mm} \]

\[ M = \frac{288 \times 0.51^2 \times 2.5}{2} = 93.636\text{kNm} \]

\[ A_{st} = \frac{1.15 \times 93.636 \times 10^6}{0.78 \times 513 \times 415} = 648\text{mm}^2 \]

Alternatively

\[ A_{st} = A_{st} \left[ \frac{2my}{L_{mx}} \right] \left[ \frac{d}{d_1} \right] \left[ \frac{510}{1025} \right] \left[ \frac{525}{513} \right] = 651.6\text{mm}^2 \]

Provide 6 numbers of 12Ø \( A_{st} = 678\text{mm}^2 \).
Shear failure at sections of beams and cantilevers without shear reinforcement will normally occur on plane inclined at an angle 30° to the horizontal. If the angle of failure plane is forced to be inclined more steeply than this [because the section considered (x – x) in Fig 24/IS456:2000 is close to a support or for other reasons], the shear force required to produce failure is increased.

The enhancement of shear strength may be taken into account in the design of sections near a support by increasing design shear strength of concrete, to \((2d_\tau/a_v)\) provided that design shear stress at the face of support remains less than the values given in Table 20/IS456:2000. Account may be taken of the enhancement in any situation where the section considered is closer to the face of a support of concentrated load than twice the effective depth, \(d\). To be effective, tension reinforcement should extend on each side of the point where it is intersected by a possible failure plane for a distance at least equal to the effective depth, or be provided with an equivalent anchorage.

14.2 Shear reinforcement for sections close to supports

If shear reinforcement is required, the total area of this is given by:

\[
A_s = a_v b (\tau_c - 2d \tau_c/a_v) / 0.87 fy > or = 0.4a_v b / 0.87 fy
\]

This reinforcement should be provided within the middle three quarters of \(a_v\). Where \(a_v\) is less than \(d\), horizontal shear reinforcement will be effective than vertical.

14.3 Enhanced shear strength near supports (simplified approach)

The procedure given 14.1 and 14.2 may be used for all beams. However for beams carrying generally uniform load or where the principal load is located farther than 2\(d\) from the face of support, the shear stress may be calculated at a section a distance \(d\) from the face of support. The value of \(\tau_c\) is calculated in accordance with Table 6 and appropriate shear reinforcement is provided at sections closer to the support, no further check for shear at such sections is required.

15.1 General

In structures where torsion is required to maintain equilibrium, members shall be designed for torsion in accordance with 15.2, 15.3 and 15.4. However, for such indeterminate structures where torsion can be eliminated by releasing redundant restraints, no specific design for torsion is necessary provided torsional stiffness is neglected in the calculation of internal forces. Adequate control of any torsional cracking is provided by the shear reinforcement as per 40/IS456:2000.

Note: The approach to design in this clause for torsion is as follows:

Torsional reinforcement is not calculated separately from that required for bending and shear. Instead the total longitudinal reinforcement is determined for a fictitious bending moment which is a function of actual bending moment and torsion; similarly web reinforcement is determined for a fictitious shear which is a function of actual shear and torsion.

15.1.1 The design rules laid down in 15.3 and 15.4 shall apply to beams of solid rectangular cross-section. However, these clauses may also be applied to flanged beams by substituting \(b_w\) for \(b\), in which case they are generally conservative; therefore specialist literature may be referred to.
15.2 Critical Section

Sections located less than a distance \(d\), from the face of the support may be designed for the same torsion as computed at a distance \(d\), where \(d\) is the effective depth.

15.3 Shear and Torsion

15.3.1 Equivalent shear
Equivalent shear, \(V_e\) shall be calculated from the formula:
\[
V_e = V_u + 1.6 \frac{T_u}{b}
\]
Where
- \(V_e\) = Equivalent shear,
- \(V_u\) = Shear
- \(T_u\) = Torsional moment, and
- \(b\) = Breadth of beam

The equivalent nominal shear stress, \(\tau_{ve}\), in this case shall be calculated as given in 40.1/IS456:2000, except for substituting \(V_u\) by \(V_e\). The values of \(\tau_{ve}\) shall not exceed the values of \(\tau_c\) max given in Table 20/IS456:2000.

15.3.2 If the equivalent nominal shear stress \(\tau_{ve}\) does not exceed \(\tau_{c}\)' given in Table 19/IS456:2000, minimum shear reinforcement shall be provided as specified in 26.5.1.6/IS456:2000.

15.3.3 If \(\tau_{ve}\) exceeds \(\tau_c\), given in Table 19, both longitudinal and transverse reinforcement shall be provided in accordance with 15.4.

15.4 Reinforcement in Members Subjected to Torsion

15.4.1 Reinforcement for torsion, when required, shall consist of longitudinal and transverse reinforcement.

15.4.2 Longitudinal Reinforcement

The longitudinal reinforcement shall be designed to resist an equivalent bending moment, \(M_{el}\) given by
\[
M_{el} = M_u + M_t
\]
Where,
- \(M_u\) = bending moment at the cross-section, and
- \(M_t\) = \(T_u((1+D/b)/1.7))\), where \(T_u\) is the torsional moment, \(D\) is the Overall depth of the beam and \(b\) is the breadth of the beam.

15.4.2.1 If the numerical value of \(M_t\) as defined in 15.4.2 exceeds the numerical value of the moment \(M_u\), longitudinal reinforcement shall be provided on the flexural compression face, such that the beam can also withstand an equivalent moment \(M_{e2}\) given by \(M_{e2} = M_l - M_u\), the moment \(M_{e2}\) being taken as acting in the opposite sense to the moment \(M_u\).

15.4.3 Transverse Reinforcement

Two legged closed hoops enclosing the corner longitudinal bars shall have an area of cross-section \(A_{sv}\) given by
\[
A_{sv} = \frac{T_u S_v}{b_1 d_1 (0.87f_y)} + \frac{V_u S_v}{2.5d_1 (0.87f_y)}
\]
but the total transverse reinforcement shall not be less than
\[
(\tau_{ve} - \tau_{vc}) b \cdot S_v
\]
\[
\frac{0.87f_y}{b}
\]
Where,
- \(T_u\) = Torsional moment,
- \(V_u\) = Shear force
- \(S_v\) = Spacing of the stirrup reinforcement,
\( b_{1} = \) Centre to center distance between corner bars in the direction of the width,
\( d_{1} = \) Center to center distance between corner bars in the direction of the depth,
\( b = \) Breadth of the member,
\( \tau_{ve} = \) Equivalent shear stress as specified in 15.3.1.
\( \tau_{c} = \) Shear strength of the concrete as specified in Table 19.
CHAPTER 17.0 SHALLOW FOUNDATIONS

General Notes

- When the resultant of the load deviates from the Center line by more than 1/6 of its least dimension at the base of footing it should be suitably reinforced.

- Unreinforced foundation may be of concrete (or) Stone masonry (or) brick masonry provided that angular spread of load from the pier/column/bed plate to the outer edge of the ground bearing is not more than 1 vertical to ½ horizontal for masonry and 1 vertical to 1 horizontal for Cement Concrete.

- The minimum thickness of the foundation at the edge should not be less than 150 mm for footings on soil and 300 mm above the top of piles for footings on piles. In case the depth to transfer the load to the ground bearing is less than the permissible angle of spread, the foundations should be reinforced.

- If the allowable / Safe bearing capacity is available only at greater depth, the foundation can be rested at a higher level for economic considerations and the difference in level between the base of foundation and the depth at which the allowable bearing capacity occurs can be filled up with either

  - (a) Concrete of allowable compressive strength not less than the allowable bearing pressure.
  - (or)

  - b) In compressible fill material, for example, sand, gravel, etc. in which case the width of the fill should be more than the width of the foundation by an extent of dispersion of load from the base of the foundation on either side at the rate of 2 vertical to 1 Horizontal.

In the case of plain Concrete pedestals, the angle between the plane passing through the bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane shall be governed by the expression.

\[
\tan \alpha \text{ not less than } 0.9 \sqrt{100 q_o + 1} \\frac{f_{ck}}{q_o}
\]

Where,

- \( q_o \) = Calculated maximum bearing pressure at the base of the pedestal in N/mm\(^2\).
- \( f_{ck} \) = Characteristic strength of concrete at 28 days in N/mm\(^2\).

- In the case of footings on piles, computation for moments and shears may be leased on the assumption that the reaction from any pile is concentrated at the Centre of the pile.

- For the purpose of computing stresses in footings which support a round or octagonal concrete column or pedestal, the face of the column or pedestal, shall be taken as the side of a square inscribed within the perimeter of the round or octagonal column or pedestal.

Eg.:-

- \( q_o = 0.2 \text{ MPa} \)
- \( f_{ck} = 20 \text{ MPa} \)
- \( \tan \alpha = 0.9 \sqrt{100q_o + 1} \)
\[ f_{\text{k}} \]
\[ = 0.9 \times \sqrt{20 + 1} = 0.9 \times 1.4142 \]
\[ = 1.2728 \]
\[ \alpha = 51.8^\circ \]
\[ \alpha \] Shall not be less than this value.

The greatest bending moment to be used in the design of an isolated Concrete footing which supports a column, pedestal or wall, shall be the moment computed at sections located as follows.

1) At the face of the column, pedestal or wall, for footings supporting a concrete column, pedestal or wall.
2) Halfway between the centre-line and the edge of the wall, for footings under masonry walls, and
3) Half way between the face of the column or pedestal and the edge of the gusseted base, for footings under gusseted bases.

17.2 Shear and bond:

The shear strength of footings is governed by more severe of the following two conditions.

1) Footing acting essentially as a wide beam, with a potential diagonal crack extending in a plane across the entire width; the critical section for this condition shall be assumed as a vertical section located from the face of the column, pedestal or wall at a distance equal to the effective depth of footing for footings on piles.

2) Two-way action of the footing, with potential diagonal cracking along the surface of truncated cone or pyramid around the concentrated load; in this case; the footing shall be designed for shear in accordance with appropriate provisions.

17.3 Tensile reinforcement:

The tensile reinforcement at any section shall provide a moment of resistance at least equal to the bending moment on Section calculated.

1) In one-way reinforced footing, the reinforcement extending in each direction shall be distributed uniformly across the full width of the footing.

2) In two-way reinforced square footing, the reinforcement extending in each direction shall be distributed uniformly across the full width of the footing; and

3) In two-way reinforced rectangular footing, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing. For reinforcement in the short direction, a Central band equal to the width of the footing shall be marked along length of the footing and
portion of the reinforcement determined in accordance with the equation given below shall be uniformly distributed across the Central band.

\[
\text{Reinforcement in Central band width} \quad \frac{2}{\beta + 1} = \text{Total reinforcement in short direction}
\]

Where \( \beta \) is the ratio of the long side to the short side of the footing. The remainder of the reinforcement shall be uniformly distributed in the outer portions of the footing.

**17.4 Transfer of load at the base of column:**

The Compressive stress in concrete at the base of the column or pedestal shall be considered as being transferred by bearing to the top of the supporting pedestal or footing. The bearing pressure on the loaded area shall not exceed the permissible bearing stress in direct compression multiplied by a value equal to

\[ \sqrt{\frac{A_1}{A_2}} \]

but not greater than 2.

Where,

- \( A_1 \) = This is the supporting area for bearing of footing. In sloped or stepped footing this may be taken as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal; and
- \( A_2 \) = loaded area at the column base.

For working stress method of design, the permissible bearing stress on full area of concrete shall be taken as 0.25 \( f_c \); for limit state method of design the permissible bearing stress shall be 0.45 \( f_c \).

Where the permissible bearing stress on the concrete in the supporting or supported member would be exceeded, reinforcement shall be provided for developing the excess force, either by extending the longitudinal bars into the supporting member, or by dowels.

**17.5 Spread foundations:**

The load on the column is spread over a considerable area of earth by means of foundation footing; so that the intensity of load on soil is kept well within the safe bearing pressure of the Soil.

Maximum settlement should not exceed 25 mm and the differential settlements of a structure should not exceed 20 mm.

**Table 17.1 Approximate safe bearing capacity in kN/Sq. m.**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Type of Soil</th>
<th>Approximate safe bearing capacity in Tonnes / Sq.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Alluvial Soil, made ground, Very wet sand</td>
<td>5</td>
</tr>
<tr>
<td>02</td>
<td>Soft Clay, Wet or loose sand</td>
<td>10</td>
</tr>
<tr>
<td>03</td>
<td>Ordinary dry clay, dry fine sand</td>
<td>20</td>
</tr>
<tr>
<td>04</td>
<td>Firm dry clay</td>
<td>30</td>
</tr>
<tr>
<td>05</td>
<td>Compact sand or gravel, hand compact clay</td>
<td>40</td>
</tr>
</tbody>
</table>
17.6 Some footing Types

(i) Spread footing

(ii) Sloped footing

(iii) Stepped footing

Note: Dimensions are indicative.

Limit State design of footing

The design of footing consists of two parts.

Part 1: Size and depth which depends on the soil characteristics.

Part 2: Structural design.

Size of foundation

\[ q = \frac{P}{A} \]

\( q \) = bearing pressure on the soil (kN/m\(^2\))

\( P \) = unfactored axial load on the foundation (kN)

\( A \) = Area of the foundation = \( B \times L \) (m\(^2\))
B = width of foundation (m)
L = Length of foundation (m)

Maximum pressure on the soil = \( \frac{P}{A} \)

**Size of foundation with moment**

\[ q = \frac{P}{A} \pm \frac{M_y}{I} \]

q = bearing pressure on the soil (kN/m²)
P = unfactored axial load on the foundation (kN)
A = Area of the foundation = B × L (m²)
M = Bending moment (kNm)
I = M.I of base area = BL³ / 12 (m³)
(Moment is about an axis parallel to B)

Y = distance of point under consideration from C.G of base (m)
B = width of foundation (m)
L = Length of foundation (m)

Maximum pressure on the soil = \( \frac{P}{A} \pm \frac{M_y}{I} \)

Minimum pressure on soil = \( \frac{P}{A} \pm \frac{M_y}{I} \)

Where \( y = \frac{L}{2} \) in both the cases.

Minimum pressure shall be \( \geq 0 \) for no tension condition

**Depth of foundation**

The depth of foundation (D) is measured from the ground level to the bottom surface of the lean concrete (P.C.C). The depth of the foundation depends on the nature of the soil.

The following are the guide lines.

1.0. \( D \geq 400\)mm in rocky soil
\( D \geq 1000\)mm in clay on sandy soil

2.0 Soil filling, sloping soil requires careful examination to rest the foundation.

3.0 In case of depths of adjacent foundations having different reduced levels, the difference between levels, should not exceed half the clear spacing of the foundation slabs in sandy or clayey soils and it should not exceed the clear spacing in rocky soils.

**17.7 Design steps of a square footing:**
17.7.1 Side of a square footing:

Total load = W kN
Side of column = b mm
Safe Bearing capacity (S.B.C.) = p kN/ Sq.m.
Area required = \( \frac{W}{P} \) Sq.m.
Side of square footing = \( \sqrt{\frac{W}{P}} \) m

17.7.2 Bending moment:

The footing is acting as a Cantilever loaded by the soil reaction.

The B.M. at the face of the Column is due to the reaction of the soil on the trapezium hatched in the figure below:
Area = \( \left( \ell - b \right) \times \left( \ell - b \right) \times \frac{1}{2} \)
\[ = \left( \ell - b \right)^2 \]
Moment = \( \left( \ell - b \right)^3 \times \frac{p}{24} \)
There are two such triangles: \( p \times 2 \times \left( \ell - b \right)^3 = \left( \ell - b \right)^3 \times p \)

\[ \text{BM / M width} = \frac{2}{3} \left( \ell - b \right) / 2 \text{ kNm per meter width.} \]

17.7.3 Bending:

The depth of footing and area of steel per meter width is calculated based on this B.M.
The required area of steel for total B.M. is provided in a length equal to the effective width of the footing where the effective width of footing
\[ d = \text{effective depth of footing}. \]

**17.7.4 Punching:**

The depth of footing at the face of the Column must be sufficient to prevent the column punching through the footing.  
Area of footing excluding area of column = \((l^2 - b^2)\)  
Area resisting punching = periphery of column \(\times\) effective depth = \((4b \times d)\)  
Punching Shear Stress = \(\frac{(l^2 - b^2) \times p}{4bd}\)  
This must be within the permissible punching shear stress which is 1.5 times the permissible Shear Stress.

**17.7.5 Shear:**

The Section of the footing must be sufficient to resist shear at a distance of effective depth from the face of the Column.

**17.7.6 Bond stress:**

The bond stress at the face of the column must be within the allowable value.  
\[ P \ (l^2 - b^2) = \text{force} \]  
Area = 4 \(d \Sigma_0\)  
\(\Sigma_0\) is the perimeter of all the bars provided in one direction.  
It is desirable that the base width of the Column is such that the portion of load pressing direct to the soil under the Column that is \((p - b^2)\) must be at least half the total load on the Column.

**13.6 Design steps for rectangular footing for rectangular column.**

(a) Intensity at base = \(q = \frac{W}{(a \times b)}\)  
Equating the upward force and downward punching force,  
\[ q (axb - cxd) = 2 (c +d) \times d_1 \times s. \ d_1 = \text{effective depth}. \]
should be less than the permissible punching shear (Which is equal to $0.16\sqrt{f_{ck}}$)

Limit state $0.25\sqrt{f_{ck}}$

(a) **Ordinary diagonal shear**

Upward force $= W/a^2 \left( a^2 - e^2 \right)$ .......... (1)

Area resisting $4e \times d$  

Force resisting $= 4e \times d \times s$ .......... (2)

$d = W \left( a^2 - e^2 \right) / \left( a^2 \times 4e \times d \times s \right)$ Equating (1) and (2) we get $d$.

$s = \text{ permissible punching shear which is equal to maximum permissible shear stress.}$

(b) **Bending moment due to cantilever action of the footing projection 'p'**

BM $= (aq^2 / 2)$ on unit width of foundation block.

To find 'd' 

Thickness of the edge should be not be less than 150 mm for footing on soils not less than 300 mm above the top of piles for footings on piles.

$Mu/bd^2 = \text{ Limit state value (Table of SP:16)}$

Calculate provided $d$

Then find $Mu/bd^2$ and $pt$

From $pt$ ; find $Ast$

**Example 17.1:**

**Design a sloped square footing**

*for a short column with axial load 1500 kN (unfactored) safe bearing capacity of soil: 200 kN / sq.n.*

*Use steel Fe =415 MPa and concrete mix M20*

To find size of column

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$ (Clause 39.3 of IS: 456:2000)
As per Clause 26.5.3.1 of IS : 456:2000, minimum % of reinforcement is 0.8 % and the maximum is 6%. However it is advisable not to exceed 4% for case of concreting. Let % reinforcement be 2%.

Let the gross area be \( A_c \).

\[ A_{sc} = 0.02 \ A_c \quad \text{and} \quad A_C = 0.98 \ A_c \]

Hence \( P_u = 0.4 \ f_{ck} \times 0.98 \ A_c + 0.67 \ f_y \times 0.02 \ A_c = A_c \ (0.392f_{ck} + 0.0134 \ f_y) \)

Factored load = \( 1.5 \times 1500 = 2250 \) KN

\[ 2250 = A_c \ (0.392f_{ck} + 0.0134 \ f_y) = A_c \ (0.392 \times 20 + 0.0134 \times 415) = 13.401 \ A_c \]

\[ A_c = \left( \frac{2250 \times 1000}{13.401} \right) = 167898 \ mm^2 \]

Providing square column \( a = \sqrt{167898} = 409.75 \ mm \)

Provide 520 x 520mm size column

**Footing**

Unfactored load is considered to arrive at the size of footing since the same is arrived at using safe bearing capacity.

Unfactored load = 1500 kN

Generally, 10 % of the load acting is added towards self weight of the footing. Footing replaces the soil. Hence the difference in self weight of soil replaced by footing and that of footing is only to be added.

Safe bearing capacity = 200 kN /m²

\[ \frac{1500}{200} = 7.5 \ sqm \]

Size = \( \sqrt{7.5} = 2.739 \ m \)

Provide 2.75 m x 2.75 m footing,

\[ q_u = \left( \frac{1.5 \times 1500}{2.75^2} \right) = 297.5 \ kN /m^2 = 0.2975 \ N/mm^2 \]

**Design of footing**

\[ \frac{2750 + 520}{2} \times 1115 = 1823025 \ sqmm \]

Soil reaction on this = \( 0.2975 \times 1823025 = 542350 \) N = 542.35 kN

C.G. of this area from the face of column

\[ = \frac{2 \times 2750 + 520}{3} = 684 \ mm = 0.684 \ m \]

Moment at the face of column = \( (542.35 \times 0.684) \) kNm

\[ M_u = 371.1 \ kNm \]

Let \( d \) be the effective depth for balanced section.

\[ x_{u \ max} = 0.48 \ d \]

\[ M_u \ limit = 0.36 \ f_{ck} \ b \times x_{u \ max} \ (d - 0.42 \ x_{u \ max}) = 0.138 \ f_{ck} \ bd^2 \]

\[ 371.1 \times 10^6 = 0.138 \times 20 \times 520 \ d^2 \]

\[ d = \sqrt{(371.1 \times 10^6 / 0.138 \times 20 \times 520)} = 512 \ mm \]

Over all depth = 512 +40+ 8 = 560 mm
Depth at free end = 150 mm  (Minimum as per Clause 34.1.2 of IS:456:2000)

\[ M_u = 371.1 \times 10^6 \]

\[ \frac{M_u}{bd^2} \]

\[ = \frac{371.1 \times 10^6}{520 \times 512^2} \]

\[ = 2.72 < 2.76 \]

\[ p_t \ (for \ M20 \ fe \ 415) = 0.937 \ (Table \ 2 \ SP: \ 16) \]

\[ A_{fl} = \frac{0.937 \times 520 \times 512}{100} = 2495 \ mm^2 \]

This is to be provided for a width of 2750 mm

\[ \frac{2495}{2750} \]

\[ = 0.90 \]

Minimum reinforcement = \[ \frac{2495}{100} \]

\[ = 61.4 \ mm^2 \]

Provide @ 120 mm c/c giving an area of 942 mm^2.

Check one way shear

Depth of section \[ D_1 = 150 + \frac{560 - 150}{1115} \times 603 = 372 \ mm \]

\[ d_1 = 372 - 40 - 8 = 324 \ mm \]

Top width at this section = \[ 2 \times 512 + 520 = 1024 + 520 = 1544 \ mm \]

Average width = \[ \frac{2750 + 1544}{2} = 2147 \ mm \]

\[ M_u = 0.2975 \times 2750 \times 603 \times (603/2) = 148.74 \ kNm \]

Refer clause 40.1.1

\[ M_u \]

\[ V_u \ ( + \ or \ - ) \ \\
\]

\[ \frac{\tan \beta}{d} \]

\[ \tau_v = \frac{V_u ( + or - )}{bd} \]

\[ \frac{560 - 150}{1115} = 0.3677 \]

\[ V_u = 2750 \times 603 \times 0.2975 = 493.3 \ KN \quad (q_u = 0.2975 \ N/mm^2) \]

\[ 148.74 \times 10^6 \\
493.3 \times 10^3 \ ( + or - ) \]

\[ \frac{324}{0.3677} \]

\[ \tau_v = \frac{2147 \times 324}{493.3 \times 10^3 \ ( + or - ) \times 168802} \]

\[ \frac{2147 \times 324}{0.466 \ N/mm^2 \ or \ 0.95 \ N/mm^2} \]

\[ = 0.466 \ N/mm^2 \]

\[ p_t \ @ \ section \ considered = \frac{2147 \times 324}{942} = 0.29 \% \]

\[ \frac{1000 \times 324}{0.39 \ Mpa < \tau_v . \ Hence \ the \ depth \ is \ inadequate.} \]

Let us increase the depth to 750 mm

\[ d = 750 - 40 - 8 = 702 \ mm \]

\[ M_u \]

\[ 371.1 \times 10^6 \]

\[ \frac{371.1 \times 10^6}{520 \times 702^2} \]

\[ = 1.45 \; \; \; p_t = 0.443 \]
\[
A_{ui} = \frac{0.443 \times 520 \times 70^2}{100} = 588 \text{ mm}^2
\]
\[
\text{Minimum required} = \frac{100 \times 1000 \times 702}{0.12} = 842 \text{ mm}^2
\]

Provide 16 mm dia bars @ 120 mm c/c giving 1675 mm\(^2\)

\[
p_t = \frac{1675 \times 100}{1000 \times 702} = 0.24
\]

\[
\tau_c = 0.33 + \frac{1000 \times 702 \times 0.04}{0.1} = 0.354 \text{ MPa}
\]

At the free end, let the over all depth be = 200 mm; \(d = 200-40-8 = 152 \text{ mm}\)

\[
\tan \beta = \frac{1115}{1924} = 0.49
\]

\[
V_u = 0.2975 \times 2750 \times 413 / 1000 = 337.9 \text{ kN}
\]

\[
M_u = 0.2975 \times \frac{702 - 152}{2 \times 10^6} = 69.77 \text{ kNm}
\]

Depth @ the section = 152 + \(-\frac{702 - 152}{1115}\times x 413 = 356 \text{ mm}\)

Width = 2 x 702 + 520 = 1404 + 520 = 1924 mm

\[
\eta_v = \frac{337.9 \times 10^3\times (\pm)^{69.77 \times 10^6 \times 0.49}}{1924 \times 356} = 0.35 \text{ MPa}
\]

\[
\tau_c = 0.354 > 0.35 \text{ Mpa.}
\]

Hence satisfactory

**Check for two way shear**

The critical section is at a distance \(d\) from the face of column (ie) at 351 mm

Critical section side width = 2 x 351 + 520 = 1222 mm

Perimeter = 4 x 1222 = 4888 mm

Depth = 152 + \(-\frac{1115 - 351}{1115}\times x 413 = 529 \text{ mm}\)

Area of concrete resisting two way.

Shear = 4888 x 529 = 2585752 mm\(^2\)

\[
\text{Two way shear force (punching shear) on the critical section}
\]

\[
0.2975 \times (2750^2 - 1222^2) = \frac{1805.6 \times 1000}{1000} = 1805.6 \text{ kN}
\]

Nominal shear stress = \(\tau_v = \frac{1805.6 \times 1000}{2585752} = 0.7 \text{ MPa}\)

Shear stress shall not exceed = 0.25 x 20 = 1.12 MPa (Clause 31.6.3.1 of IS:456:2000)

Hence satisfactory

**Check for bond length**

\[
L_d = (\varphi \sigma_s / 4 \tau_{bd}) = (\varphi \times 0.87 f_y / 4 \tau_{bd})
\]

\[
\tau_{bd} = 1.2 \times 1.6 = 1.92 \text{ N/mm}^2 \text{ (Clause 26.2.1.1 of IS:456:2000)}
\]

\[
0.87 \times 415 \times 16
\]
Ld = \[ \frac{4 \times 1.92}{1.5} = 752 \text{ mm} \]

Available length = 1115 – cover = 1115 - 50 = 1065 mm > 752mm
Bond length is sufficient.

**Example 17.2:**

**Square footing for a short Column with factored axial load 1500kN. S.B.C. 200 kN/Sq.m. Steel Fe 415 & Concrete M20.**

**Column design:**

Reinforcement can be between 0.8 % to 4%

\[ P_u = 0.4 \times fck \times A_c + 0.67 \times f_y \times A_{sc} \] (Clause 39.3)

\[ 1500 \times 1000 = 0.4 \times 20 \times A_c + 0.67 \times 415 \times A_{sc} \]
\[ 15 \times 10^3 = 8 \times A_c + 278.05 \times A_{sc} \]
\[ A_{sc} = 0.01 \times A_g \]
\[ A_c = 0.99 \times A_g \]
\[ A_g = 140180 \text{ mm}^2 \]

Side of square Column = 374.4 mm Provide 375 mm

\[ A_{at} \] required = \[ 0.01 \times 375 \times 375 \] = 1406 mm²

Provide 8 bars of 16 dia. \[ A_{st} = 1608 \text{ mm}^2 \] use 10 mm lateral tie.

Volume contained in 1 set of lateral tie.

\[ = (4 \times 295 \times 78) + (4 \times 208 \times 78) = 562500 \text{ mm}^3 \]

Volume of lateral reinforcement required for 1 m height of Column (0.4%)

\[ = 0.4 \times 375 \times 375 \times 1000 = 562500 \text{ mm}^3 \]

100

Spacing of lateral ties \[ = 156936 \times 1000 = 279 \text{ mm} \]

562500

Provide 10 mm Φ ties @ 250 mm c/c

**Square footing:**

Safe bearing capacity = 200kN/Sq m

Unfactored load = \[ \frac{1500}{1.5} = 1000 \text{ kN} \]

Minimum area of footing required = \[ 1000 \times 1000 = 5 \times 10^6 \text{ Sq.mm} = 5 \text{ Sqm} \]

Size : \[ \sqrt{5} = 2.24 \text{ m} \]
Provide 2.5 x 2.5m footing.

\[ q = \left( \frac{(15 \times 10^3)}{(2500 \times 2500)} \right) = 0.24 \text{ N/mm}^2 \]

Depth of footing required for punching shear @ the face of the Column.

\[ = \left( \frac{(0.24(2500^2 - 375^2))}{(4 \times 375 \times 0.25 \times \sqrt{20})} \right) = 874 \text{ mm} \]

Provide 900mm over all depth.

**Note:** Calculated shall not exceed 0.25 \( \sqrt{fck} \) (Clause 31.6.3.1)

Bending moment / m width of footing @ the face of the column.
= 0.24 \left(2500 - 375\right)^2 \times (2 \times 2500 + 375) \times 10^3 = 647.24 \text{kNm}
\frac{Mu}{24 \times 375} = 2.76 \text{bd}^2

\text{Depth of footing required} = \sqrt{\frac{(647.24 \times 10^6)}{(2.76 \times 1000)}} = 484 \text{ mm}.

\text{Provided depth} 900 \text{ mm} \text{ is adequate.}
\text{d} = 900 - 50 - 8 = 842 \text{ mm}

\text{Mu} = 647.24 \times 10^6 = 0.91
\frac{Bd^2}{1000 \times 842^2} = 0.264
\text{Pt} = 0.264 \times 1000 \times 842 = 2223 \text{mm}^2

\text{The reinforcement must be used for an effective width of} \ \frac{b + l + d}{2} \times 2 = 375 + 2500 + 842
= 2280 \text{ mm}.

\text{Therefore, Per Meter width, Ast} = 975 \text{mm}^2

\text{12 Dia @ 110mm c/c; Ast} = 1028 \text{mm}^2

\text{Provide sloping foundation with 300 mm depth @ the free end.}
\text{The effective depth at (d) 842 mm from the face of the column}
= 300 + (600/1062.5) \times 220.5 = 424.5 \text{ mm}
= 424.5 - 50 - 10 = 364.5 \text{ mm}.

\text{Shear Stress at 842 mm from the face of the Column} = 0.24 \times (2500^2 - 2059^2) = 0.16 \text{ MPa}.
4 \times 2059 \times 364.5

\text{To find allowable shear stress:-}
\text{Reinforcement 12 mm diameter Tor @ 110 C/C.}
A_{st}/.m width = 1028 \text{mm}^2
\text{Pt} = \frac{1028}{1000 \times 364.5} \times 100 = 0.28\%
\text{Design shear strength}

\text{From Table 61 of SP6(1)}

<table>
<thead>
<tr>
<th>Pt</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.33</td>
</tr>
<tr>
<td>0.3</td>
<td>0.39</td>
</tr>
</tbody>
</table>

0.33 + (0.06/0.1) \times 0.08 = 0.378 > 0.28
Hence satisfactory.

Bond Stress at the face of the column = \( \frac{0.24 \times (2500^2 - 375^2)}{(4 \times 842 \times 12 \times \pi \times 22)} \) = 0.53 N/mm²

< 0.8 MPa x 1.6 = 1.28 MPa

Hence satisfactory.

**Example 17.3:**

Design a sloped square footing for a short axially loaded column with axial load 1500 kN, (unfactored) safe bearing capacity of soil = 200 kN/m², Use steel Fe = 415 MPa and concrete mix M20.

To find the size of column:

\[
\begin{align*}
\text{Factored load} &= 1.5 \times 1500 = 2250 \text{ kN} \\
\text{Unfactored load} &= 1500 \text{ kN} \\
\text{Self weight of footing not added since it replaces soil and SBC} &= 200 \text{ kN/m}² \\
\text{Area of footing required} &= \frac{1500}{200} = 7.5 \text{ m}² \\
\text{Size} &= \sqrt{75} = 27.38 \text{ m} \\
\end{align*}
\]

Provide 520 x 520 mm size column.

Footing:

Unfactored load = 1500 kN

Self weight of footing not added since it replaces soil and SBC =200 kN/m²

Area of footing required = (1500/200) =7.5m²

Size = \( \sqrt{75} = 27.38 \text{ m} \)

Provide 2.75m x 2.75 m footing

\[
q_u = \frac{1.5 \times 1500}{2.75^2} = 297.5kN/m^2 = 0.2975N/mm^2
\]

**Design of footing:**

Shaded area = \( \frac{2750 + 520}{2} \times 1115 = 1823025 m^2 \)

Soil reaction on this = 0.2975 x 1823025 = 542350 N = 542.35 kN

C.G. of this area from the face of column =

\[
\begin{align*}
\text{Moment at the face of column} &= (542.35 \times 0.684) \text{kNm} = 371.1 \text{kNm} \\
\text{Let d be the effective depth for balanced section,} \\
x_u \lim &= 0.48d \\
M_u &= 0.36f_{ck} b x_u \lim (d - 0.42 x_u \lim) = 0.138f_{ck} bd^2 \\
371.1 \times 10^6 &= 0.136 \times 20 \times 520 \times d^2 \\
\therefore d &= \sqrt{\frac{371.1 \times 10^6}{0.136 \times 20 \times 520}} = 512 mm \\
D &= 512 + 40 + 8 = 560 mm \\
\text{Depth at free end} &= 150mm \text{ (clause 34.1.2)}
\end{align*}
\]
for M20 & Fe 415, Table 2 sp:16

This is to be provided for a width of 2750

Minimum reinforcement =

Provide 12 φ @ 120 mm c/c (942mm²)

Check one way shear:

Depth of section = D₁ =

Top width at this section = 2 x 512 + 520 = 1024 + 520 = 1544 mm

Average width =

\[
M_u = 0.2975 \times 2750 \times 603 \times \frac{603}{2} = 148.74 kNm
\]

Refer clause 40.1.1

The value is 0.47

P₁ at section considered =

Hence increase the depth.
Let us increase the depth to 750mm

d₁ = 750 - 40 - 8 = 702mm

Minimum required =

Provide 16 φ @ 120 mm c/c = 1675 mm²
At the free end; let the overall depth be = 200 mm, 
d= 200-40-8=152mm

\[\tan \beta = \frac{702-152}{1115} = 0.49\]

\[V_u = 0.2975 \times 2750 \times \frac{413}{1000} = 337.9 kN\]

\[M_u = 0.2975 \times \frac{413^2}{2} \times \frac{2750}{10^6} = 69.77 kNm\]

Depth at the section =

Width = \(2 \times 702 + 520 = 1404 + 520 = 1984\) mm

Hence satisfactory

Check for one way shear:

The critical section is at a distance \(d/2\) from the face of column (i.e.) at 351mm.

Critical section side width = \(2 \times 351 + 520 = 1222\) mm.

Perimeter = \(4 \times 1222 = 4888\) mm

Depth =

Area of concrete resisting two way shear = \(4888 \times 529 = 2585752\) mm\(^2\)
Two way shear force (Punching shear) on the critical section

\[
= 0.2975 \left( \frac{2750^2 - 1222^2}{1000} \right) = 1805.6 kN
\]

Nominal shear stress = \( \frac{1805}{25857} \) MPa

Permitted = \( \phi \) MPa

Hence satisfactory

Check for bond length

\[
L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}
\]

\[
\tau_{bd} = 1.2 \times 1.6 = 1.92 N / mm^2 (\text{clause 26.2.1.1})
\]

\[
L_d = \frac{0.87 \times 415 \times 16}{4 \times 1.92} = 752 mm
\]

Available length = 1115 – cover = 1115-50=1065mm>752mm

Bond length is sufficient
17.8 Combined footings:-

These are necessary when the external Column of a building is very near the boundary line and a Separate foundation slab can not be provided. Also when two columns are close by such that isolated footing overlaps, combined footings are provided.

1) The exterior and interior columns are provided with a combined footing so that the Centre of gravity of the Column loads is in the same vertical line as the centre of gravity of the soil reaction. To ensure this, a rectangular or trapezoidal footing is designed.
2) The bending moments on the footing slabs are calculated as for a reversed floor supported at the Columns and loaded by soil reactions.
3) If the projection in the transverse direction is fairly large, the footing has to be designed as a Cantilever.
4) Bond and shear stress at column edges and at points of zero bending moment must be examined.

Example 17.4:

Distance between center of Columns 3700 mm. Sizes of Columns are 500 x 500 mm and 600 x 600 mm and the loads are 1200 kN and 1700 kN respectively. The space available for the footing is restricted to 1800 mm in width and length can be as short as possible. SBC = 320 kN / sqm. (from soil test report). The loads are unfactored.

Total load on the soil due to two Columns = 1200 + 1700 = 2900 kN

Self weight of the footing is neglected since the footing replaces the soil and the density of soil is almost equal to the density of concrete

Required area of footing = (2900/320) = 9.0625 Sq.m.

Width = 1800 mm (Restricted)

Length required = (9.0625/1.8) = 5.034 m (Restricted)

Provide 5.1 m x 1.8 m

Distance of C.G. of loads from the 1200 kN Column

{(1700 x 3700)/2900} = 2169 mm say 2170 mm.
To coincide the C.G. of foundation and the C.G. of load, the foundation slab has to be cast as shown. Since the C.G of foundation (plan area) and the C.G of load coincides, the pressure on soil can be assumed as uniform.

The pressure on soil is

\[ q = \frac{(2900 \times 10^3)}{(1800 \times 5100)} = 0.316 \text{ N/mm}^2 \]

\[ = \frac{(0.316 \times 10^6)}{10^3} = 316 \text{ kN/Sq.m.} \]

Taking moment about face of column A =

\[ M_u = (1.5 \times 316 \times 1.02^2 / 2) = 246.6 \text{ kNm.} \]

Taking moment about face of column B =

\[ M_u = (1.5 \times 316 \times 0.38^2 / 2) = 34.22 \text{ kNm.} \]

Average – Ve BM = \((246.6 + 34.22) / 2 = 280.8 / 2 = 140.4 \text{ kN m}\)

+ve BM @ Mid Span = \((1.5 \times 316 \times 3.7^2) / 8 = 811.1 \text{ kNm}\)

Net BM at mid span = 811.1 – 140.4 = 670.73 kN.m

\[ M_u = 2.76 \text{ (Limit)} \]

\[ bd^2 \]

\[ d = \sqrt{\left(\frac{670.73 \times 10^6}{2.76 \times 1000}\right)} = 493 \text{ mm} \]
Overall depth, \( D = 493 + 50 + 12.5 = 555.5 \text{ mm} \)

Providing 750 mm over all depth; clear cover 50mm and 25mm dia bar,

\[
d = 750 - 50 - 12.5 = 687.5 \text{ mm}.
\]

\[
\frac{\mu_u}{bd^2} = \frac{670.73 \times 10^6}{1000 \times 687.5^2} = 1.42
\]

\[
Pt = 0.4345; \text{ Ast} = \frac{0.4345}{100} \times 1000 \times 687.5 = 2987 \text{ mm}^2
\]

Provide 20 \( \varphi \) @ 100mm c/c; Ast = 3142mm²

\[
\frac{\mu_u}{bd^2} = \frac{34.2 \times 10^6}{1000 \times 687.5^2} = 1.42
\]

\[
Pt = 0.12 \text{ (Min)}
\]

Minimum \( A_{st} \) = 0.12% = \( \frac{0.12}{100} \times 687.5 \times 1000 = 825 \text{ mm}^2 \)

Provide 12 \( \varphi \) @ 130mm c/c

\( A_{st} \) required for – ve BM @ face of Column A

\[
\frac{\mu_u}{bd^2} = \frac{246.6 \times 10^6}{1000 \times 687.5^2} = 0.52
\]

\[
Pt = 0.15
\]

\[
Ast = \frac{0.15}{100} \times 1000 \times 687.5 = 1031 \text{ mm}^2
\]

Provide 16 \( \varphi \) @ 140 mm c/c (Ast = 1436mm²)

Projection of slab from face of Column in the transverse direction.

\[
1800 / 2 = 900 \text{ mm} ; \ 0.9 \text{ m}
\]

\[
\mu = (1.5 \times 316 \times 0.9^2 / 2) = 192 \text{ kN m}
\]

\[
d = 687.5 - 12 = 675.5 \text{ mm}.
\]

\[
\frac{\mu_u}{bd^2} = \frac{192 \times 10^6}{1000 \times 675.5^2} = 0.42
\]
Pt = 0.12

Provide 12 φ @ 130 mm c/c

**To find point of Contra flexure**

Shear force @ middle of column B = 1.8 \times 0.38 \times 1.5 \times 316 = 324.2 kN

Max shear force = 1681 kN

Shear Stress = \frac{1681 \times 10^3}{1800 \times 687.5} = 1.36 \text{ MPa}

1436 \times 100 = 0.21\%

Pt @ this Point = 1000 \times 687.5

Shear Stress Permissible (τ_c) = 0.33 \text{ MPa (Table 61, SP:16)}

Shear Capacity of Concrete = \frac{0.33 \times 1800 \times 687.5}{1000} = 408 \text{ kN}

Balance for which stirrups are to be provided = 1681 – 408 = 1273 kN

\( V_{us} = \frac{1273}{68.75} = 18.52 \) d

Provide 6 legged 12 φ @ 130mm c/c stirrups (Table 62, SP: 16)

\[
\frac{A_{S_v}}{b S_v} \geq \frac{0.4}{0.87 f_z} \\
\frac{6 \times 113}{1800 \times S_v} \geq \frac{0.4}{0.87 \times 415} \\
\frac{6 \times 113 \times 0.87 \times 415}{1800 \times 04} = 340 \text{ mm}
\]
Hence 12 φ six legged @ 130 mm c/c is satisfactory.

**17.9 Design of pedestal:-**

Steel Column subjected to an axial force of 750 kN (factored) is mounted on concrete pedestal. Size of Base Plate is 360 x 620 mm. SBC = 120 kN/m² @ 1.5 M below G.L.

Load on Column = 750 kN
Size of base plate = 360 x 620 mm.
Depth of foundation = 1500 mm
Height of Pedestal above plinth = 300 mm
Concrete grade M20
Reinforcement $f_y$ = 415 MPa.

Size of Pedestal (having 75 mm concrete around base plate) = $(360 +150) \times (620 +150)$

$$= 510 \times 770$$

Min. Asc = 0.15 % (clause 26.5.3.1 (h)) = $0.0015 \times 510 \times 770 = 589 \text{ mm}^2$

Provide 6 Nos. of 12 φ
Asc provided = $6 \times 113 = 678 \text{ mm}^2 > 589 \text{ mm}^2$

**Capacity:-**

$Pu = 0.4 \times f_{ck} \times A_c + 0.67 \times f_y \times A_{sc}$

$$= 0.4 \times 20 \times (510\times770 - 678) + 0.67 \times 415 \times 678$$

$$= 3324.7 \text{ kN} > 750 \text{ kN}$$

Diameter of tie = $1/4^{th}$ of main bar = 12/4 = 3mm

However provide 8 φ ties.

Spacing < Least lateral dimension (or) 300mm whichever is less spacing = 300mm

**Design of footing slab:-**

Area of foundation required = $Af = \frac{Po}{Pa} = \frac{750}{120} = 6.25 \text{ m}^2$

Ratio of Pedestal size: $(770 / 510) = 1.5$

Width = $b$; Length = $l = 1.5 \times b$

Area = $1.5 \times b^2 = 6.25$ (maintaining the same ratio)

$$h = \sqrt{6.25 / 1.5} = 2.04 \text{ m}$$

Provide 2.1 m x 3.1m

$A = 2.1 \times 3.1 = 6.51 \text{ Sq. m.}$

$q = (\frac{750}{6.51}) = 115.2 \text{ kN/m}^2 = 0.115 \text{ MPa}$

Projection = $(3100 - 770)/2 = 1165 \text{ mm}$

Bending moment in the footing slab: $(0.115 \times 1165^2) / 2 = 78040 \text{ N.mm.}$
Thickness of footing slab at the face of the pedestal =

\[ M_u = 2.76 \text{ (Limit)} \]
\[ \frac{bd^2}{d} = \sqrt{\frac{78040}{2.71 \times 1}} = 168 \text{ mm} \]

Provide over all depth as 300 mm and 150 mm at free end.

Effective depth = 300 – 40 – 10 = 250 mm

\[ M_u = \frac{78040}{1 \times 250^2} = 1.25 \]
\[ Pt = 0.376 \text{ (Table 2, SP: 16)} \]
\[ Ast \text{ required} = \left(\frac{0.376}{100}\right) \times 1000 \times 250 = 940 \text{ mm}^2 \]

Provide 12 φ @ 120 mm c/c

\[ Ast = 942 \text{ mm}^2 \]

**Reinforcement on the other direction:-**

\[ M = \frac{(0.115 \times 795^2)}{2} = 36341 \text{ N mm} \]
\[ d = 250 - 16 = 234 \text{ mm} \]

\[ M_u = \frac{36341}{1 \times 234^2} = 0.66 \]
\[ Pt = 0.187 \]
\[ Ast = (0.187/100) \times 1000 \times 234 = 437.6 \text{ mm}^2 \]

Provide 12 φ @ 250 mm c/c Ast 452 mm²

**Check for shear:-**

Critical Shear is at a distance d from the face of the pedestal. (Two way shear)
$510 + 2 \times 250 = 1010\ mm$

$770 + 2 \times 250 = 1270\ mm$

Punching Shear @ the critical plane.

$V = (2100 \times 3100 - 1010 \times 1270) \times 0.115 = 601\ kN$

$150 + \left\{(300 - 150)/1165\right\} \times (1165 - 250) = 150 + 118 = 268\ mm.$

$d = 268 - 40 - 8 = 220\ mm.$

Area of Surface $= 2(1010 + 1270) \times 220 = 1003200\ mm^2$

Shear Stress $= (601 \times 1000 / 1003200) = 0.6\ MPa.$

Allowable Shear Stress in punching $= 0.25 \sqrt{f_{ck}}$

$= 0.25 \sqrt{20} = 1.12\ MPa > 0.6\ MPa.$

Hence Satisfactory
Example 17.5:

The columns spaced at 4550mm (c/c) are subjected to 400kN and 500kN superimposed loads respectively. The size of each column is 450mm square. The total length of the foundation is to be restricted to 5000mm i.e., outer to outer faces of the column. The safe bearing capacity of the soil is 60kN/m². Use M20 mix concrete and $F_e = 415\text{MPa}$.

Total load = $400 + 500 = 900\text{kN}$ (Unfactored)

SBC = $60\text{kN/m}^2$

Let us assume that the difference between the weight of concrete be about 10% of total load.

Hence total load = $900 + 90 = 990\text{kN}$ (Unfactored)

To find the C.G of load, taking moment about 400kN column.

$\frac{500 \times 4550}{400 + 500} = 2528$ from centre of column carrying 400kN.

The C.G of the base of foundation is at middle of column centre lines as shown (C.G.B). The C.G of loads is at 2528 mm from centre line of column carrying 400kN.

There is an eccentricity of $2753 - 2500 = 253\text{mm}$.

$M = 900 \times \frac{253}{1000} = 227.7\text{ kNm}$

'To find required width

Pressure on soil = $q = \frac{P}{A} \pm \frac{M}{Z}$
$Z = \text{Section modulus of the base about } zz \text{ axis. Since eccentricity is about } zz \text{ axis}$

\[ Z = \frac{BD^2}{6} = \frac{B \times 5000^2}{6} \]

Area = $B \times 5000$

\[ \therefore \frac{P}{A} + \frac{M}{Z} = \frac{990 \times 1000}{B \times 5000} + \frac{227.7 \times 10^6 \times 6}{B \times 5000^2} \neq 60 \times 10^{-3} \text{ N/mm}^2 \text{ (SBC)} \]

\[ \frac{1}{B}(198 + 54.65) = \frac{252.65}{B} = 60 \times 10^{-3} \]

\[ \therefore B = \frac{252.65}{60} \times 10^3 = 4211 \text{ mm.} \]

Providing $B = 4500 \text{ mm}$,

\[ \frac{P}{A} + \frac{M}{Z} = \frac{990 \times 1000}{5000 \times 4500} + \frac{227.7 \times 10^6 \times 6}{4500 \times 5000^2} \]

\[ = 0.044 + 0.01214 \]

\[ = 0.056 \text{ N/mm}^2. \]

\[ = 56 \text{ kN/m}^2 < 60 \text{ kN/m}^2 \text{ (SBC)} \]

Minimum pressure on soil

\[ \frac{P}{A} - \frac{M}{Z} = \frac{990 \times 1000}{5000 \times 4500} - \frac{227.7 \times 10^6 \times 6}{4500 \times 5000^2} \]

\[ = 0.044 - 0.01214 \]

\[ = +0.032 \]

Since $\frac{P}{A} > \frac{M}{Z}$, there is no tension in the soil
Design of foundation slab

Pressure at column face:

\[ q = 0.032 + \left( \frac{0.056 - 0.032}{5000} \right) \times 4550 = 0.054 \text{MPa} \]

Factored = 1.5 x 0.054 = 0.081 MPa

Projection = 2025 mm
\[ M_u = \frac{0.081 \times 2025^2}{2} = 0.166 \times 10^6 \text{kNm} \]

Using M20 mix and Fe 415 reinforcement
\[ \frac{M_u}{bd^2} = \frac{0.166 \times 10^6}{b \times d^2} = 2.76 \]
\[ d = \frac{M_u}{b \times 2.76} = 245.24 \text{ mm} \]

Shear will decide the depth. It is better to consider higher depth. Hence keeping overall depth as 500mm.
\[ d = 500 - 50 - 8 = 442 \text{ mm} \]
\[ \frac{M_u}{bd^2} = \frac{0.158 \times 10^6}{b \times 442^2} = 0.81 \]
\[ P_t = 0.236; A_{st} = \frac{0.236 \times 1000 \times 442}{100} = 1043 \text{ mm}^2 \]

For 1m width \( A_{st} = 1043 \text{ mm}^2 \)

Provide 16\( \phi \)@190mm c/c 1058 mm²

Critical section for shear is at a distance equal to effective depth from face of column.

Effective depth at critical section = \( d_1 \)
\[ = 150 + \frac{(500 - 150)}{2025} \times 1583 = 423.60 \text{ mm} \]
\[ p_t = \frac{1058 \times 100}{1000 \times 423.60} = 0.249, \tau_c = 0.36 \text{ Mpa (table 61 of SP:16)} \]

Shear force = \( \frac{1000(2025 - 442) \times 0.081}{1000} \) = 128.223kN
\[ \tau = \frac{128.223 \times 1000}{1000 \times 423.60} = 0.2887 < 0.36 \text{ hence satisfactory.} \]

Distribution steel = \( \frac{0.12 \times 1000 \times 442}{100} \) = 530mm²

Provide 10\( \phi \) @ 140mm c/c 561mm²

Design of beam

The beam connecting the columns can be designed as an overhang beam with varying load as a T section. The location where moment will be max can be found where shear force is zero. Here an approximate method neglecting overhang is followed to find the maximum B.M.

UDL @ A = 48x4.5 = 216kN/m
UDL @ B = 84x4.5 = 378 kN/m
S.F=0

\[ 216 \times x + x \left( \frac{378 - 216}{1} \right) \frac{1}{2} = 400 \times 1.5 \]
\[ x = 2.36 \text{ m}^2 \]
\[M = 216 \times \frac{2.36^2}{2} + \frac{(378 - 216) \times 2.36}{5} \times \frac{2.36}{2} \times \frac{2.36}{3} - 400 \times 1.5(2.36 - 0.225) = 608.50\]

Maximum bending moment = 608.50 kNm

Thickness of slab at free end = 150mm.
Thickness of slab at face end = 500mm.
Average thickness = \(\frac{650}{2} = 375\)mm.

Slab will act as flange to T-section

Effective width of flange = \(b_f = \frac{L}{3} = \frac{4550}{3} = 1517\)mm

Refer 2.4 of SP:16

\[\frac{D_f}{d} = \frac{150}{442} = 0.34 > 0.2\]

\[\frac{b_f}{b_w} = \frac{1517}{450} = 3.37\]

Refer Table 58

For \(\frac{D_f}{d}=0.34 \& \frac{b_f}{b_w}=3; \frac{M_u,lim}{b_w d^2 f_{ck}} = 0.361\)

For \(\frac{D_f}{d}=0.34 \& \frac{b_f}{b_w}=4; \frac{M_u,lim}{b_w d^2 f_{ck}} = 0.473\)

\[= 0.361 + \frac{(0.473-0.361)}{1} \times 0.37 = 0.402\]

\[\frac{M_u,lim}{b_w d^2 f_{ck}} = 0.402\]

\[\therefore M_u,lim = 0.402 \times 1517 \times 442^2 \times 20/10^6 = 2383 \text{ kNm} > 608.50 \text{ kNm}\]

\[\frac{M_u}{b_w d^2 f_{ck}} = \frac{608.50 \times 10^6}{1517 \times 442^2 \times 20} = 0.102\]

\[p_i = 0.085\]

\[A_{st} = \frac{0.085 \times 450 \times 442}{100} = 169 \text{ mm}^2\]

Minimum \(A_{st}\)

\[A_{st} = \frac{0.85}{fy} \times b d\]

\[A_{st} = \frac{0.85}{415} \times 450 \times 442 = 407 \text{ mm}^2\]

Provide 4nos 12 \(\phi\); Ast = 452 mm²

\[\frac{x_w}{d} = \frac{0.87 \times 415 \times 452}{0.36 \times 20 \times 1517 \times 442} = 0.033 < 0.48\]

(Refer 9.1.1 of SP: 16)

\[M.R = \frac{0.87 \times 415 \times 452 \times 442}{10^6} \left( 1 - \frac{452 \times 415}{1517 \times 442 \times 20} \right) = 71 \text{ kNm} < 610.43 \text{ kNm}\]

Increase the reinforcement

Provide 10nos 25\(\phi\) Ast = 4908 mm²

\[M.R = \frac{0.87 \times 415 \times 4908 \times 442}{10^6} \left( 1 - \frac{4908 \times 415}{1517 \times 442 \times 20} \right) = 783 \times 0.85 = 664 \text{ kNm} > 610.43 \text{ kNm}\]

Hence satisfactory

Now \(\frac{x_u}{d} << 0.48\)

(Table B / SP: 16 for \(f_y=415\))

Design for shear:

Critical section for shear force is at a distance $d$ from face of a column.

This section from mid point is

$$2250 - \frac{450}{2} - d = 2025 - 442 = 1583\text{mm}$$

$$S.F = 216 \times 0.667 + \frac{1}{2} \times 0.667 \left( \frac{100}{5} \times 0.667 \right) - 600$$

$$\tau = \frac{739.3 \times 1000}{1517 \times 442} = 1.10\text{ MPa} < 2.8\text{ MPa}$$

Hence section need not be increased

Considering that 2nos 25$\phi$ are granted and 8nos 25$\phi$ are taken through;

$$p_t = \frac{3926 \times 100}{1517 \times 442} = 0.585$$

$$\tau_c = 0.51\text{MPa}$$

Shear resistance of concrete $= \frac{0.51 \times 1517 \times 442}{1000} = 342\text{ kN}$

Shear to be resisted by cranked bars and stirrups $= 739.3 - 342 = 397.3\text{ kN}$

Shear to be resisted by 2nos 25$\phi$ cranked $= 2 \times 125.32$

$$= 250.64\text{ kN}$$

(Refer Table-63 of SP: 16)

Balance to be restricted by stirrups $= 146.7\text{ kN}$

$$\frac{V_{us}}{d} = \frac{146.7}{44.2} = 3.32\text{ kN/cm}$$

8 # 4 legged @ 200 c/c. $\left( \frac{V_{us}}{d} = 2 \times 1.815 = 3.63 \right)$
I.2 Design of Isolated footing subjected to axial load and bending

A column is subjected to an axial force of 1000kN and bending moment of 140kNm. The size of the column is 450mm × 600mm. SBC is 120kN/m²

Design of foundation

Load on soil = 1000 + 10% of 1000kN
= 1100kN

The maximum pressure on the soil is limited to the allowable bearing capacity of soil

\[ \frac{P}{A} \leq \frac{SBC}{Z} \]

\[ A = B \times L \]

\[ Z = \frac{B^2 L}{6} \]

We have to assume either B or L and find the other one.

Approximate method to find one dimension

Considering axial load only, with square footing;

size required \( a = \sqrt{\text{load/area}} = \sqrt{\frac{1100}{120}} = 3.03 \text{m} \)

Keep \( B = 3 \text{m} \)

To find \( L \)

\[ \frac{1100}{3 \times L} + \frac{140}{(3^2 \times L)/6} = \frac{366.67}{L} + \frac{93.33}{L} + \frac{460}{L} = 420 \]

\[ \therefore L = \frac{460}{120} = 3.83 \text{m} \]

Let \( L = 4 \text{m} \)

\[ P + \frac{M}{A} = \frac{1100}{3 \times 4} + \frac{140}{(3^2 \times 4)/6} = 115 \text{kN/m}^2 < 120 \text{kN/m} \]

For no tension to develop

\[ P - \frac{M}{A} \geq 0 \]

\[ \frac{1100}{3 \times 4} - \frac{140}{(3^2 \times 4)/6} = 68.33 \text{ kN} \]
Design of the section

\[ q_3 = \frac{P}{A} + \frac{M}{1/Y} = \frac{1100}{3 \times 4} + \frac{140}{(3 \times 4^3) / 12 / 0.3} \]

\[ = 91.7 + 2.625 = 94.325 \text{ kN/m}^2 \]

Moment at face of the column

\[ M = \frac{94.325 \times 1.7^2}{2} \]

\[ = 136.3 + 20 \times 3 = 468.9 \text{ kNm} \]

\[ M_u = 1.5 \times 468.9 = 703.35 \text{ kNm} \]

Alternative

\[ M_u = 1.5 \times \frac{B L^2 m}{6} \]

\[ = \frac{1.5 \times 3 \times 1.7^2}{6} (2 \times 115 + 94.325) = 702.97 \text{ kNm} \]

The moment capacity of a trapezoidal section is
\[ M_r = (kN - kN_1 b_1 + b_2 kN_1) d^2 fck \]
\[ = (0.138 - 0.025) \times 600 + 3000 \times 0.025) d^2 \times 20 \]
\[ = 1500 - d^2 \]

Equating \( M_r = M \)
\[ d = \sqrt{\frac{703.35}{1500}} = 0.6847m = 685mm \]

\( D = 685 + 40 + 6 = 731mm \)

Providing 750mm depth; \( d = 750 - 46 = 704mm \)

**Check for shear capacity – diagonal tension**

\[ b_o = 2 \left( \frac{450 + \frac{704}{2} \times 2 + 600 + \frac{704}{2} \times 2}{4916} \right) = 4916mm \]

The shear force acting on this plane is

\[ d_1 = 150 + \left( \frac{600}{1700} + \frac{704}{2} \right) = 150 + 475.8 = 625.8mm \]

Effective depth = 625.8 – 46 = 579.8mm

\[ \tau_v = \frac{V}{b_o \times d} = \frac{1312 \times 1000}{4916 \times 579.8} = 0.46N/mm^2 \]

Allowable shear stress for M20 concrete is

\[ \tau_a = kN \tau_c = 0.25 \sqrt{20} = 1.12N/mm^2 \quad 0.46N/mm^2 \]
The section is safe against shear failure including transverse shear.

**Design of reinforcement**

\[
A_{st} = \frac{1.15M}{fy} = \frac{1.15 \times 703.35 \times 10^2}{0.798 \times 704 \times 415} = 3469 \text{mm}^2
\]

Provide 12 numbers of 20\(\phi\) bars.  \(A_{st}\) provided = 3768\(\text{mm}^2\)

**The reinforcement in the short span direction**

Effective cantilever span = \(Ly = \frac{3.0 - 0.45}{2} = 1.275\text{m}\)

\(d = 704 - 20 = 684\text{mm}\)

The area of the reinforcement in the short span is

\[
A_{sy} = A_{st} = \frac{3469 \times 1.275 \times 0.704}{1.700 \times 0.684} = 2678\text{mm}^2
\]

Provide 9 numbers 20\(\phi\) \(A_{st}\) provide = 28.26\(\text{mm}^2\).

**II Combined footing for unequally loaded two columns and uniform in width**

Two columns carrying 300kN and 450kN are spaced 2m apart. The foundation is resting on soil having a SBC of 100kN/m\(^2\). The width of footing must be restricted to 2m. The column size is 450 \(\times\) 450 mm each.

\(P_1 = 300\text{kN}; P_2 = 450\text{kN}\)

\(Ps = 300 + 450 = 750\text{kN}\)

SBC = 100kN/m\(^2\)

Distance between the loads = 2m

\(f_{ck} = 20\text{MPa} \quad f_y = 415\text{MPa}\)

**Design of the foundation**

Self weight 10% 75kN

Total 750 + 75 = 825kN

\[
L = \frac{\text{Load}}{\text{Width} \times \text{SBC}} = \frac{825}{2 \times 100} = 4.125\text{m}
\]

Providing 2m \(\times\) 4.5m footing;

\[
q = \frac{825}{2 \times 4.5} = 91.7\text{kN/m}^2 \text{(unfactored)} < 100\text{kN/m}^2 \text{ safe.}
\]

The centroid of the loads must coincide with the centroid of the footing.

**Design of the section**

The cantilever spans which are to be considered for maximum bending moments at the face of the column are

\(L_a = 1050 - \frac{450}{2} = 1050 - 225 = 825\text{mm}\)

\(L_b = 1450 - \frac{450}{2} = 1450 - 225 = 1225\text{mm}\)

**Maximum cantilever moment**

\[
1.5 \times q \times B \times L_a^2 = 1.5 \times 91.7 \times 2 \times 1.45^2
\]
Moment at centroid

\[ M_2 = 1.5 \times \left( 91.7 \times 2 \times \frac{2.25^2}{2} - 300 \times 1.2 \right) = 156.35 \text{kNm (Factored)} \]

\[
d = \sqrt{\frac{M_2}{B fck}} = \sqrt{\frac{(156.35 \times 10^6)}{0.138 \times 2000 \times 20}} = 168 \text{mm}
\]

Providing over all depth of 300mm

\[ d = 300 - 40 - 6 = 254 \text{mm} \]

Design for transverse shear

The critical section is at a distance \( d \) from the face of the column and shear span is

\[ L_s = L_b - 225 - 254 = 1450 - 479 = 971 \text{mm} \]

The nominal shear stress is

\[ \tau_v = \frac{V_u}{B_d} = \frac{1.5 \times 2 \times 0.971 \times 1.17}{2 \times 0.971} = 137.55 \text{kN/m}^2 = 0.137 \text{N/mm}^2 \]

Shear capacity for minimum reinforcement (≤ 0.25%) is 0.36N/mm\(^2\) which is more than the calculated shear stress (cl 40.2.1, 40.2.2, 40.3, 40.4, 40.5.3, 41.3.2, 41.3.3 of IS: 456/2000)

Check for diagonal tension (Punching shear)

The critical shear plane of punching through diagonal tension is at 0.5d from the face of this column. The peripheral length of the column is

\[ b_o = 4 \times (a + d) = 4 \times (450 + 254) = 4 \times 704 = 2816 \text{mm} \]

The maximum shear force occurs around column B and it is

\[ V = 1.5q \left( \frac{L \times B}{2} \right)^2 \left( a + d \right)^2 \left( \frac{4.5 \times 2}{a} \right)^2 \left( 0.45 + 0.254 \right)^2 = 550.8 \text{kN} \]

The shear stress = \( \frac{550.8 \times 1000}{2816 \times 254} \) = 0.77N/mm\(^2\)

The shear strength for diagonal failure is 0.25 \( \sqrt{20} \) = 1.118N/mm\(^2\) > 0.77 kN/m\(^2\)

Hence satisfactory.

Reinforcement design

The area of tensile reinforcement at the bottom in the longitudinal direction is

\[ A_{st} = \frac{1.15M_i}{j d f_y} = \frac{1.15 \times 289.2 \times 10^6}{0.798 \times 254 \times 415} = 3954 \text{mm}^2 \]

Provide 20 bars of 16mm \( \phi \) \( A_{st} \) provided = 4021mm\(^2\)

On the other direction

Projection on the other side = \( \frac{2000 - 450}{2} = \frac{1550}{2} = 775 \text{m} \)

\[ 1.5 \times 91.7 \times 0.775^2 \]
M = \frac{2}{1.15 \times 41.3 \times 10^6} = 41.3 \text{kNm} \\
A_{st} = \frac{1.15 \times 41.3 \times 10^6}{0.798 \times 238 \times 415} = 603 \text{mm}^2/\text{m} \\
For \ 4.5 \text{m} = 4.5 \times 603 = 2712 \text{mm}^2 \\
Provide 14 number 16 \varphi A_{st} provided = 2814 \text{mm}^2 \\

Note: 
- 1.15 is the partial safety factor applied to steel in flexural stress. 
- \( j = 1 - 0.42 \times \frac{x_u}{d} = 1 - 0.42 \times 0.479 = 0.798 \) for \( f_y = 415 \text{MPa} \)

Design of cantilever beams

A portico of size 5m \times 3m is supported by two beams fixed into the column. Two cantilever beam kept 3m apart is supporting the portico 750mm height 115mm thick parapet is provided along the periphery.

Imposed load 1.5kN/m²  \( fck = 20 \text{MPa} \)  \( fy = 415 \text{MPa} \)
Thickness of slab = \frac{3000}{25} = 120 \text{mm} \\
Weight of slab = 1 \times 1 \times 0.12 \times 25 = 3.00 \text{kN/m}^2 \\

Design of slab

Finish load 1kN/m² \\
Total load/m² = 1.5 + 3.0 + 1.0 = 5.5 \text{kN/m}^2 \\
W_u = 1.5 \times 5.5 = 8.25 \text{kN/m}^2 \\
Weight of the parapet = 0.115 \times 0.75 \times 20 = 1.725 \text{kN/m} \\
W_{cu} = 1.5 \times 1.725 = 2.5875 \text{kN/m} \\

\[ M_{u1} = \frac{8.25 \times 1^2}{2} = 2.5875 \times 1 = 4.125 + 2.5875 = 6.71 \text{kNm} \]

\[ \text{Mid span moment} = \frac{8}{8.25 \times 3^2} M_{u1} \]
Design of the section for maximum of the two
\[ d = \sqrt{(6.71 \times 10^6 / 0.138 \times 1000 \times 20)} = 49.3\text{mm} \]

Provide overall depth = 115mm
\[ d = 115 - 15 - 5 = 95\text{mm} \]

For mid span moment of 2.57kNm;
Provide 8 \( \phi \) at 285mm c/c since the spacing shall not exceed 3d (i.e. 3 \times 95 = 285mm) (Refer Table 15)
For support moment 6.71kNm; referring to table 15, 8 \( \phi \) at 300 c/c giving 168mm\(^2\) is required. Alternate bars from mid span will be oranted which will be at 570mm c/c.
Hence provide additional bars of 8 \( \phi \) at 570mm thus giving the spacing as 285mm.

**Distribution of steel**
\[ \frac{0.15 \times 1000 \times 95}{100} = 142.5\text{mm}^2 \]
Provide 8 \( \phi \) at 350mm c/c

**Design of beam**

Span depth ratio for cantilever beam = 7
\[ \frac{3000}{7} = 430\text{mm} \]
Provide size of beam 230 \times 450mm
Depth of web = 450 - 115 = 335mm
Self wt = 0.23 \times 0.335 \times 25 \times 1.5 = 2.89kN/m

Load from slab = 8.25 \times (3 / 2 + 1) 8.25 \times 2.5 = 20.625kN/m
Wt of Parapet = 0.115 \times 0.75 \times (3 / 2 + 1) \times 20 \times 1.5 = 6.47kN/m
Weight of Parapet parallel to beam = 0.115 \times 1 \times 20 \times 1.5 = 2.59kN/m
Total udl = 2.89 + 20.625 + 2.59 = 26.105kN/m

\[ Mu = \frac{26.105 \times 3^2}{2} = 6.47 \times 3 = 117.47 \times 19.41 = 136.88\text{kNm} \]

Let \( b = 300\text{mm} \)
\[ Mu = \frac{136.88 \times 10^6}{300 \times d^2} = 2.76 \quad d = \sqrt{\frac{136.88 \times 10^6}{300 \times 2.76}} = 406.7\text{mm} \]

\[ D = 406.6 + 25 + 8 = 440\text{mm} \]
Providing \( D = 450\text{mm} \); \( d = 450 - 33 = 417\text{mm} \)

\[ Mu = \frac{26.105 \times 3^2}{2} = 6.47 \times 3 = 117.47 \times 19.41 = 136.88\text{kNm} \]

Pt = 0.892 from table 2 of fIS: 456 design aids
Design for Shear

\[ V = 26.105 \times 3 + 6.47 = 84.785 \text{kN} \]

\[ \tau_v = \frac{84.785 \times 1000}{300 \times 417} = 0.68 \]

\[ pt = \frac{300 \times 417}{300 \times 417} \times 100 = 1.00 \]

\[ \tau = 0.62 \text{MPa} < 0.68 \text{MPa} \]

Stirrups are required.

\[ V_c \text{ Shear Capacity} = \tau_c bd = \frac{0.62 \times 300 \times 417}{1000} = 77.56 \text{kN} \]

S.F for which shear is to be provided

\[ V_{us} = V - V_c = 84.785 - 77.56 = 7.225 \text{kN} \]

\[ \frac{V_{us}}{D} = \frac{7.225}{41.7} = 0.17 \text{kN/cm} \]

Provide 2 legged 8, \( A_{sv} = 100.6 \text{mm}^2 \)

\[ S_v = \frac{0.87 \times 100.6 \times 415 \times 417}{7225} = 2096 \text{mm} \]

Minimum spacing of the stirrup is

\[ S_{max} = \frac{A_{sv} f_y}{0.4u \times 0.4 \times 300} = \frac{100.6 \times 415}{348 \text{mm}} \]

\[ = 0.75 \times d = 0.75 \times 417 = 312.75 \text{mm} \]

Provide 2 legged 8 \( \phi \) at 300mm/c.

Cross Section of slab

CHAPTER 18 LIMIT STATE DESIGN OF REINFORCEMENT CONCRETE STRUCTURES

\( A_{st} = 0.892 \times 300 \times 417 / 100 = 1115.9 \text{mm}^2 \)

Provide 4 – 20 \( \phi \) \( A_{st} = 1256 \text{mm}^2 \)
18.1 Members subjected to axial compression and bending.

Factored axial load in the beam = P_u kN
Center to center of column =
Span of beam = L_c metres
Effective length of column = 0.65 L_c (Depending on end condition)
Breadth of beam = b mm
Over all depth = D mm
Ratio of effective length to least lateral dimension = (0.65 L_c/b)
If 0.65 L_c/b < 12 the column is a short column.
Po = P_u kN
If 0.65 L_c/b > 12 the column is a long column and hence load reduction factor is to be applied. (Cr)
Cr = 1.25 – 0.65 L_c/48 b
Equivalent short column load = Po = P_u/Cr kN (maximum)
M_uh = M_us kNm (maximum)
M_uh is the greater value of
M_us
Area of steel provided for hogging moment = A_sreg = A_st
Diameter of main steel for hogging moment @ support, f mm
No. of rods required for column action = A_sreg / A_f1 = N_ca (rounded off to next higher integer.
If A_sreg < A_st, provide nominal shear reinforcement otherwise shear reinforcement is to be provided in the beam to take up hogging and sagging moments there rods are to be provided thro’ out the section.

18.2.1 Design for shear (IS:456:2000-Clause 40 and Table 20)

Shear stress \( \tau_s \) in MPa (SP:16 cl.4.2) = \( V_u \times 1000/b_w \times d \)
if \( f_y = 20 \) MPa \( \tau_{c_{max}} = 2.8 \) MPa
if \( f_y = 25 \) MPa \( \tau_{c_{max}} = 3.1 \) MPa
if \( f_y = 30 \) MPa \( \tau_{c_{max}} = 3.5 \) MPa
if \( f_y = 35 \) MPa \( \tau_{c_{max}} = 3.7 \) MPa
if \( f_y = 40 \) MPa \( \tau_{c_{max}} = 4.0 \) MPa
If \( \tau_s > \tau_{c_{max}} \) (actual shear stress exceeds the maximum shear stress), size of section will be increased and redesigned.
Design shear strength of concrete \( \tau_c \) is as in Table 19 of IS:456:2000.
Shear capacity of concrete section \( V_c = (f_c b_d/1000) \) kN
If \( V_c > V_u \) provide nominal shear reinforcement otherwise shear reinforcement is to be provided.
Shear to be carried by stirrups = \( V_{us} \) kN = \( V_{us} = (V_u - V_c) \) kN
Diameter of bar proposed to be used for stirrups f c mm
No. of legs \( N_a \)
Area of vertical legs = \( A_{sv} = N_a \times p \times f_2 2/4 \) sq mm
Spacing of stirrups \( S_v \) in mm (IS 456:2000 Clause 40.4)
= 0.87 \( f_y \) \( A_{sv} \times d/V_{us} \times 1000 \). Check whether the spacing of stirrups \( S_v \) less than or equal to lesser of the following.
  (g) \( S_v = 0.75 d \) IS 456:2000 Clause 26.5.1.5
  (h) \( S_v = 300 \) mm
  (i) \( S_v = A_{sv} \times 0.87 f_y/0.4 \) b
\( S_v \) the lowest of all the above. Provide f2 mm diameter \( N_a \) legged stirrups @ \( S_v \) mm spacing.

18.2.2 Development length and anchorage

Stress in steel = \( ss \) MPa (SP:16 5.1 of Page 183)
\( ss = 0.87 f_y \) MPa
for developing full strength in the bar
if $f_{ck} = 20$  $\eta_{bd} = 1.2 \times 1.6 = 1.92$
if $f_{ck} = 25$  $\eta_{bd} = 1.4 \times 1.6 = 2.24$
if $f_{ck} = 30$  $\eta_{bd} = 1.5 \times 1.6 = 2.40$
if $f_{ck} = 35$  $\eta_{bd} = 1.7 \times 1.6 = 2.72$
if $f_{ck} = 40$  $\eta_{bd} = 1.9 \times 1.6 = 3.04$

$L_d = f_t \times s_j/4 \eta_{bd} = \text{mm}$  (IS 456:2000-Clause 26.2.1)

**Abstract**

**size of beam**

Breadth ‘b’ mm =
Over all depth ‘D’ mm =
Clear cover 25 mm =
Reinforcement to take up tension $f_{te}$ mm dia at a spacing of $S_v$ mm.
Development length of bar $L_d$ mm.